Code : 211303

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MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *All questions carry equal marks.*

(ii) *There are* ***NINE*** *questions in this paper.*

(iii) *Attempt* ***FIVE*** *questions in all.*

(iv) *Question No.* ***1*** *is compulsory.*

1. Choose the correct answer (any *seven*) :

(a) Which of the following is an entire solution?

(i) $\frac{z}{1+z^2}$

(ii) $z\bar{z}$

(iii) e^{-z^2}

(iv) $e^{z^{-2}}$

(b) The value of $\int_C \frac{dz}{z+2}$, $C : |z| = 1$ is

(i) $2\pi i$

(ii) $-2\pi i$

(iii) $4\pi i$

(iv) 0

(c) $J_{\frac{1}{2}}(x)$ is given by

(i) $\sqrt{\frac{2\pi}{n}} \sin x$

(ii) $\sqrt{\frac{2\pi}{n}} \cos x$

(iii) $\sqrt{\frac{\pi}{2n}} \cos x$

(iv) $\sqrt{\frac{2}{\pi n}} \sin x$

(d) The polynomial $2x^2 + x + 3$ in terms of Legendre polynomial is

(i) $\frac{1}{3}(4P_2 - 3P_1 + 11P_0)$

(ii) $\frac{1}{3}(4P_2 + 3P_1 - 11P_0)$

(iii) $\frac{1}{3}(4P_2 + 3P_1 + 11P_0)$

(iv) $\frac{1}{3}(4P_2 - 3P_1 - 11P_0)$

(e) In the equation $P_0 y'' + P_1 y' + P_2 y = 0$; $x = a$ is singular point, if

(i) $P_0 = 0$

(ii) $P_0 \neq 0$

(iii) $P_1 = 0$

(iv) $P_1 \neq 0$

(f) The solution of $z(x, y)$ of the equation $\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$ is

(i) $f(x + \log_e y, z) = 0$

(ii) $f(y + \log_e x, z) = 0$

(iii) $f(x + \log_e z, y) = 0$

(iv) $f(z + \log_e x, y) = 0$

(g) The particular integral of $(D^2 - D'^2)z = \cos(x + y)$ is

(i) $\frac{x}{2}\sin(x + y)$ (ii) $x\sin(x + y)$

(iii) $x\cos(x + y)$ (iv) $\frac{x}{2}\cos(x + y)$

(h) The partial differential equation from $z = (a + x)^2 + y$ is

(i) $z = \frac{1}{4}\left(\frac{\partial z}{\partial x}\right)^2 + y$

(ii) $z = \frac{1}{4}\left(\frac{\partial z}{\partial y}\right)^2 + y$

(iii) $z = \left(\frac{\partial z}{\partial x}\right)^2 + y$

(iv) $z = \left(\frac{\partial z}{\partial y}\right)^2 + y$

(i) The probability of getting a king when 1 card is drawn from a pack of 52 cards is

(i) $\frac{4}{13}$

(ii) $\frac{1}{3}$

(iii) $\frac{8}{13}$

(iv) $\frac{9}{52}$

(j) A coin is tossed 6 times in succession. The probability of getting at least one head is

(i) $\frac{63}{64}$

(ii) $\frac{3}{32}$

(iii) $\frac{1}{64}$

(iv) $\frac{1}{2}$

2. (a) What is a singular point? Find regular singular point of the equation

$$2x^2y'' + 3xy' + (x^2 - y)y = 0$$

(b) Solve in series the differential equation

$$3xy'' + 2y' + y = 0$$

3. (a) Prove :

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx = \begin{cases} 0 & m \neq n \\ \dfrac{2}{2n+1} & m = n \end{cases}$$

(b) Explain in terms of Legendre polynomials the expression :

$$x^4 + x^3 + x^2 + x + 1$$

4. (a) Form the partial differential equation from $ax^2 + by^2 + z^2 = 1$.

(b) Solve :

$$(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$$

5. (a) By separation of variables, solve

$$2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$$

(b) Find the solution of

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

subject to boundary conditions

$$y(0, t) = 0,\ y(l, t) = 0,\ y(x, 0) = \phi(x);$$

$$\frac{\partial y}{\partial t}(x, 0) = \psi(x)$$

6. (a) What are the necessary conditions for a function $f(z)$ to be analytic, where $f(z) = 2xy + i(x^2 - y^2)$?

(b) Find the point where the function $f(z) = |z|^2$ is differentiable.

7. (a) Discuss the Cauchy integral formula and hence find the value of

$$\int_C \frac{2z^2 + z}{z^2 - 1}\,dz$$

where C is circle of unit radius with centre at $z = 1$.

(b) Find first 3 terms of Taylor series expansion of $f(z) = \dfrac{1}{z^2 + 4}$ about $z = -2$. Also find the region of convergences.

8. (a) Establish a relation between moment about mean and moment about any point.

Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means equal to 160 inches^2 and 91 inches^2 respectively. Can these be regarded as drawn from the same normal populations?

9. *(a)* Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials.

(b) The probability that a man aged 60 will live to be 70 is 0·65. What is the probability that out of 10 men now aged 60 at least 7 will live to be 70?

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Code : 211303

B.Tech. 3rd Semester Exam., 2013

MATHEMATICS—III

Time : 3 hours

Full Marks : 70

Instructions :

(i) *The questions are of equal value.*

(ii) *There are* ***NINE*** *questions in this paper.*

(iii) *Attempt any* ***FIVE*** *questions.*

(iv) *Question No.* **1** *is compulsory.*

1. Choose the correct answer of any *seven* out of *ten* :

(a) The value of integral $\int_{k}^{2-i} z\,dz$ is

(i) 0

(ii) 1

(iii) $1+2i$

(iv) $1-2i$

(b) The value of complex integral $\int_{c} \frac{z}{z^2+1}\,dz$, where c is a closed curve $|z+i| = 0 \cdot 5$, is

(i) πi

(ii) 0

(iii) $2\pi i$

(iv) $-\pi i$

(c) The value of $J_2(x)$ in terms of $J_1(x)$ and $J_0(x)$ is

(i) $2J_1(x) - xJ_0(x)$

(ii) $\frac{4}{x} J_1(x) - J_0(x)$

(iii) $\frac{2}{x} J_1(x) - \frac{2}{x} J_0(x)$

(iv) $\frac{2}{x} J_1(x) - J_0(x)$

(d) If $\int_{-1}^{1} P_n(x)\,dx = 2$, then n is

(i) 1

(ii) 0

(iii) -1

(iv) None of the above

(e) In the functions $Q_1(x) = (x-a)\,P_1(x)$ and $Q_2(x) = (x-a)^2\,P_2(x)$, if Q_1 and Q_2 are analytic, thus $x = a$ is called

(i) ordinary singular point

(ii) irregular singular point

(iii) regular singular point

(iv) None of the above

(f) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(i) $z = f_1(y + x) + f_2(y - x)$

(ii) $z = f_2(y + x) + f_1(y - x)$

(iii) $z = f_2(y + x) + f_2(y - x)$

(iv) $z = f(x^2 - y^2)$

(g) The solution of $3x\frac{\partial z}{\partial x} - 5y\frac{\partial z}{\partial y} = 0$ is

(i) $f(x^3 y, z^5) = 0$

(ii) $f(x^3 y^3, z) = 0$

(iii) $f(xy, z) = 0$

(iv) $f(x^5 y^3, z) = 0$

(h) The solution of $z = p + q$ is

(i) $f(x + y, y + \log_e z) = 0$

(ii) $f(x \cdot y, y \log_e z) = 0$

(iii) $f(x - y, y - \log_e z) = 0$

(iv) None of the above

(i) An unbiased coin is tossed 3 times. The probability of obtaining two heads is .

(i) $\frac{1}{2}$

(ii) $\frac{3}{8}$

(iii) 1

(iv) $\frac{1}{8}$

(j) The probability that a marksman will hit a target is given as $\frac{1}{5}$. Then his probability of at least one hit in 10 shots is

(i) $1 - \left(\frac{4}{5}\right)^{10}$

(ii) $\frac{1}{5^{10}}$

(iii) $1 - \frac{1}{5^{10}}$

(iv) None of the above

2. (a) Solve by Frobenius method, the differential equation $xy'' + y' + x^2 y = 0$. Indicate the form of second solution which is linearly independent of the first obtained above.

(b) Prove :

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

3. *(a)* Prove :

(i) $nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$

(ii) $(x^2-1)P_n = (n+1)(P_{n+1} - xP_n)$

(b) Show :

$$\int_{-1}^{1} x^3 \cdot P_3(x)dx = \frac{4}{35}$$

4. *(a)* Form the partial differential equation from $2z = (ax+y)^2 + b$.

(b) Solve :

(i) $y^2 p - xyq = x(z-2y)$

(ii) $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ for $z(1, y) = \cos y$

5. *(a)* By separation of the variables, solve $\frac{\partial u}{\partial x} + u = \frac{\partial u}{\partial t}$, if $u = 4e - 3x$ for $t = 0$.

(b) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

under the condition $u = 0$, when $x = 0$ and $x = \pi$; $\frac{\partial u}{\partial t} = 0$, when $t = 0$ and $u(x, 0) = x$, $0 < x < \pi$.

  (Turn Over)

6. *(a)* What are the sufficient conditions for a function $f(z)$ to be analytic? Test the analyticity of $\frac{1}{(z-1)(z+1)}$.

analytic at all point except z = ±1.

(b) Prove that $u = x^2 - y^2$ and

$$v = \frac{y}{x^2 + y^2}$$

are harmonic functions of $f(x, y)$ but are not harmonic conjugate.

7. *(a)* Evaluate

$$\int_c \frac{e^{2z}}{(z+1)^4} dz$$

where $|z-1| = 2$ is a circle.

(b) For the function $f(z) = \frac{4z-1}{z^4-1} + \frac{1}{z-1}$, find all Taylor or Laurent series about the centre zero.

8. *(a)* The frequency distribution of measurable characteristic varying between 0 and 2 is as under

$$f(x) = x^3, \quad 0 \le x \le 1$$
$$= (2-x)^3, \quad 1 \le x \le 2$$

Calculate the standard deviation and mean deviation about the mean.

(Continued)

(b) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0·05 :

x :	0	1	2	3	4
f :	419	352	154	56	19

Given, at 3 degree of freedom, $\chi^2_{0\cdot05} = 7\cdot82$.

9\. (a) A die is thrown 8 times and it is required to find the probability that 3 will show—

(i) exactly 2 times;

(ii) at least 7 times;

(iii) at least once.

(b) Define probability density function. A function $f(x)$ is defined as

$$f(x) = \begin{bmatrix} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0, & x > 4 \end{bmatrix}$$

Show that it is a probability density function.

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Code : 211303

B.Tech 3rd Semester Exam., 2015

MATHEMATICS—III

Time : 3 hours *Full Marks* : 70

Instructions :

(i) *The marks are indicated in the right-hand margin.*

(ii) *There are* ***NINE*** *questions in this paper.*

(iii) *Attempt* ***FIVE*** *questions in all.*

(iv) *Question No.* ***1*** *is compulsory.*

1. Choose the correct answer (any *seven*) :

2×7=14

(a) $x\{J_{n-1}(x)+J_{n+1}(x)\}$ is equal to

(i) $2J_n(x)$ (ii) $2J'_n(x)$

(iii) $2nJ_n(x)$ (iv) None of the above

(b) The value of

$$\left(\frac{1}{2^n}n!\right)\times\{d^n(x^2-1)^n/dx^n\}$$

is

(i) 0 (ii) 1

(iii) $P_n(x)$ (iv) None of the above

(c) The equation $p\tan y+q\tan x=\sec^2 z$ is of order

(i) 1 (ii) 2

(iii) 0 (iv) None of the above

(d) The maximum number of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}=0$ with the condition $y(0)=1$ is

(i) 4 (ii) 3

(iii) 2 (iv) 1

(e) The probability of drawing any one spade card from a pack of cards is

(i) 1/52 (ii) 1/13

(iii) 4/13 (iv) 1/4

(f) The solution of $p+q=z$ is

(i) $f(x+y, y+\log z)=0$

(ii) $f(xy, y\log z)=0$

(iii) $f(x-y, y-\log z)=0$

(iv) None of the above

(Continued)

(g) var $(2X \pm 3)$ is

(i) 5

(ii) 13

(iii) 4 if $\text{var}\, X = 1$

(iv) None of the above

(h) The moment generating function of geometric distribution is

(i) $p(1-qe^t)$

(ii) $p/(1-qe^t)$

(iii) $pe^t/(1-qe^t)$

(iv) None of the above

(i) The modulus and argument of the complex number $\frac{1+2i}{1-3i}$ are

(i) $\left(\frac{1}{\sqrt{2}}, \frac{3\pi}{4}\right)$ *(ii)* $\left(\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)$

(iii) $\left(\frac{1}{\sqrt{2}}, \frac{\pi}{2}\right)$ *(iv)* None of the above

(j) If $z = 1 + i$, then $1/z$ is

(i) $1 - \frac{i}{2}$ *(ii)* $\frac{1}{2} - i$

(iii) $\frac{1}{2} - \frac{i}{2}$ *(iv)* None of the above

2. Obtain the Taylor's and Laurent's series which represent the function

$$f(z) = \frac{1}{(1+z^2)(z+2)}$$

in the regions

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$ 14

3. Prove that

$$\int_{-1}^{1} P_m(x) \cdot P_n(x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$$ 14

4. Show that the function

$$f(z) = \sqrt{|xy|}$$

is not analytic at the origin, although the Cauchy-Riemann equations are satisfied at that point. 14

5. Solve the following : 7+7=14

(a) $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$

(b) $a(p+q) = z$

6. Show that $v(x, y) = -\sin x \sinh y$ is harmonic. Find the conjugate harmonic of v. 14

7. State and prove Bayes' theorem. 14

8. Solve in series, using Frobenius method, the equation

$$\frac{d^2y}{dx^2} + x^2 y = 0$$ 14

9. Let the random variable X assumes the value r with the probability law

$$P(X = r) = q^{r-1} p; \quad r = 1, 2, 3, \cdots$$

Find the moment generating function of X and hence its mean and variance. 14

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Code : 211303

B.Tech 3rd Semester Examination, 2016

Mathematics III

Time : 3 hours *Full Marks : 70*

Instructions :

*(i) There are **Nine** Questions in this Paper.*

*(ii) Attempt **Five** questions in all.*

*(iii) **Question No. 1 is Compulsory.***

(iv) The marks are indicated in the right hand margin.

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1. Choose the correct answer (any seven): $2\times7=14$

(a) If P_n is the Legendre Polynomial of first kind, then the value of $\int_{-1}^{1} xP_nP_n' \, dx$ is

(i) $\frac{2}{(2n+1)}$

(ii) $\frac{2n}{(2n+1)}$

(iii) $\frac{2}{(2n+3)}$

(iv) $\frac{2n}{(2n+3)}$

P.T.O.

(b) If J_n is the Bessel's function of first kind, then the value of $J_{\frac{1}{2}}$ is

(i) $\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} - \sin x\right)$

(ii) $\sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(iii) $\sqrt{\frac{2}{\pi x}}\sin x$

(iv) $\sqrt{\frac{2}{\pi x}}\cos x$

(c) For the differential equation $t(t-2)^2 y''+ty'+y=0$, $t=0$, is

(i) an ordinary point

(ii) a branch point

(iii) an irregular point

(iv) a regular singular point

(d) The partial differential equation $y^2u_{xx}-(x^2-1)u_{yy}=0$, is

(i) parabolic in $\{(x,y): x<0\}$

(ii) Hyperbolic in $\{(x,y): y>0\}$

(iii) Elliptic in $\{(x,y): y>0, x^2+y^2<1\}$

(iv) Parabolic in $\{(x,y): x>0\}$

(e) The solution of the equation $xp^3q^2 + yp^2q^3 + (p^3 + q^3) - zp^2q^2 = 0$ is, $z =$

(i) $ax + by + (ab^{-2} + ba^{-2})$

(ii) $ax - by + (ab^{-2} - ba^{-2})$

(iii) $-ax + by + (-ab^{-2} + ba^{-2})$

(iv) $ax + by - (ab^{-2} + ba^{-2})$

(f) Which of following is correct, where $(i = \sqrt{-1})$?

(i) $1 + i > 2 - i$

(ii) $2 + i > 1 + i$

(iii) $2 + i < 1 + i$

(iv) none of these

(g) The residue of $\frac{\sin z}{z^8}$ at z=0 is

(i) 0

(ii) 1/7!

(iii) −1/7!

(iv) none of these

(h) If A and B are two events and the probability $P(B) \neq 1$, then $\frac{P(A) - P(A \cap B)}{1 - P(B)}$ equals

(i) $P(A/\overline{B})$

(ii) $P(A/B)$

(iii) $P(\overline{A}/B)$

(iv) $P(\overline{A}/\overline{B})$

(i) If A and B are two events such that $P(A) = .3$, $P(B) = .6$ and $P\left(\frac{B}{A}\right) = .5$ then $P\left(\frac{A}{B}\right)$ is equal to

(i) $\frac{2}{5}$

(ii) $\frac{5}{8}$

(iii) $\frac{1}{4}$

(iv) $\frac{3}{5}$

(j) Let X be a continuous random variable with probability density function:

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} e^{\frac{x^2}{4}}; -\infty < x < \infty \end{cases}$$

, then its moment generating function $M_X(t)$ is 0, otherwise

(i) e^{t}

(ii) e^{t^2}

(iii) $e^{\frac{t^2}{2}}$

(iv) $e^{-\frac{t^2}{2}}$

2. Solve in series, using Frobenius method, the equation

$$x(1-x)y''+\left(\frac{3}{2}-2x\right)y'+2y=0$$ 14

3. State and prove orthogonal properties of Legendre Polynomials. 14

4. Find the analytic function $f(z)=u+iv$, where

$$u=\frac{\sin 2x}{(\cosh 2y+\cos 2x)}$$ 14

5. Solve the following $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=$

$(2x^2+xy-y^2)\sin xy-\cos xy$ 14

6. Classify and reduce the following equation

$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}+2\frac{\partial z}{\partial y}-2\frac{\partial z}{\partial y}+z=0$ into normal form and find its solution. 14

7. (a) State and prove Bay's Theorem. 7+7=14

(b) For any two events A and B, prove that

$$P(A\cap\overline{B})=P(A)-P(A\cap B)$$

8. (a) Discuss controller area network and its distributed system. Describe CAN interface for a DSP56805 processor. Also give details of all the PIN. 10

(b) Discuss the differences between strobe and handshake signals. 4

9. Write notes on the following :

(a) Memory shadowing 4

(b) DSP processor application 4

(c) AVR microcontroller architecture 6

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