

B.Tech 2nd Semester Exam., 2015

MATHEMATICS—II

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.
- (ii) There are **NINE** questions in this paper.
- (iii) Attempt **FIVE** questions in all.
- (iv) Question No. 1 is compulsory.

1. Choose the correct or best alternatives
(any seven) :

2×7=14

(a) In a series of positive terms $\sum u_n$, if

$$\lim_{n \rightarrow \infty} u_n \neq 0$$

then the series $\sum u_n$ is

- (i) convergent
- (ii) divergent
- (iii) not convergent
- (iv) oscillatory

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(b) The p -series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ to ∞ is divergent for

- (i) $p = 1$
- (ii) $p > 1$
- (iii) $p < 1$
- (iv) $p \leq 1$

(c) The series $a - a + a - a + a - \dots$ to ∞ is

- (i) convergent
- (ii) divergent
- (iii) oscillatory
- (iv) None of the above

(d) The Laplace transform of a signal $y(t)$ is

$$y(s) = \frac{1}{s(s-1)}$$

then its final value is

- (i) -1
- (ii) 0
- (iii) 1
- (iv) unbounded

(e) The Laplace transform of the function $f(t) = t$, starting at $t = a$ is

- (i) $\frac{1}{(s+a)^2}$
- (ii) $\frac{e^{-as}}{(s+a)^2}$
- (iii) $\frac{e^{-as}}{s^2}$
- (iv) $\frac{a}{s^2}$

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(f) If from the function $f(t)$ one forms the function $\psi(t) = f(t) + f(-t)$, then $\psi(t)$ is

- (i) even
- (ii) odd
- (iii) neither even nor odd
- (iv) both even and odd

(g) The triple integral $\iiint_T dx dy dz$ gives

- (i) volume of region T
- (ii) surface area of region T
- (iii) area of region T
- (iv) density of region T

(h) The double integral

$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$$

is

- (i) 0
- (ii) π
- (iii) $\pi/2$
- (iv) 2

(i) $\nabla \times (\nabla \times \vec{A})$, where \vec{A} is a vector, is equal to

- (i) $\vec{A} \times \nabla \times \vec{A} - \nabla^2 \vec{A}$
- (ii) $\nabla^2 \vec{A} + \nabla(\nabla \cdot \vec{A})$
- (iii) $\nabla^2 \vec{A} + \nabla \times \vec{A}$
- (iv) $\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(j) If $(\vec{A} \times \vec{B}) \times \vec{C} = \vec{A} \times (\vec{B} \times \vec{C})$, then

- (i) \vec{A}, \vec{B} are collinear
- (ii) \vec{A}, \vec{B} are perpendicular
- (iii) \vec{A}, \vec{C} are collinear
- (iv) \vec{A}, \vec{C} are perpendicular

2. (a) Test for convergence the series whose n th term is $n^{\log x}$.

(b) Test the series for convergence

$$1 + \frac{3}{7}x + \frac{3 \cdot 6}{7 \cdot 10}x^2 + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13}x^3 + \frac{3 \cdot 6 \cdot 9 \cdot 12}{7 \cdot 10 \cdot 13 \cdot 16}x^4 + \dots \text{ to } \infty$$

3. (a) Define absolutely and conditionally convergent series. Show that the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots \text{ to } \infty$$

is convergent but not absolutely convergent.

- (b) Find the Fourier series expansion of the periodic function of period 2π

$$f(x) = x^2, \quad -\pi \leq x \leq \pi$$

Hence find the sum of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \quad 14$$

4. (a) Find the Fourier series for $f(x)$, where

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

- (b) Find the Fourier half-range even expansion of the function

$$f(x) = -\frac{x}{e} + 1, \quad 0 \leq x \leq e \quad 14$$

5. (a) Find the Laplace transform of—

(i) $t^2 \cos at$;

(ii) $e^{-4t} \frac{\sin 3t}{t}$.

- (b) Find the Laplace transform of $\frac{1 - \cos t}{t^2}$. 14

6. (a) Show that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$$

- (b) Evaluate $\iint r^4 \cos^3 \theta \, dr \, d\theta$ over the interior of the circle $v = 2a \cos \theta$. 14

7. (a) Evaluate : akubihar.com

$$\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dx \, dy \, dz$$

- (b) Find the length of the arc of the curve $y = e^x$ from the point $(0, 1)$ to $(1, e)$. 14

8. (a) Find the directional derivatives of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

- (b) Find the magnitude of the velocity and acceleration of a particle which moves along the curve $x = 2 \sin 3t$, $y = 2 \cos 3t$, $z = 8t$ at any time $t > 0$. Find unit tangent vector to the curve. 14

(7)

9. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that

$$\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2}$$

- (b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$$

and C is the rectangle in the xy -plane
bounded by $y=0$, $x=a$, $y=b$, $x=0$. 14
