

## B.Tech 2nd Semester Exam., 2015

## MATHEMATICS—II

Time : 3 hours

Full Marks : 70

Instructions :

- (i) The marks are indicated in the right-hand margin.  
 (ii) There are **NINE** questions in this paper.  
 (iii) Attempt **FIVE** questions in all.  
 (iv) Question No. 1 is compulsory.

1. Choose the correct or best alternatives  
 (any seven) :  $2 \times 7 = 14$

(a) In a series of positive terms  $\sum u_n$ , if

$$\lim_{n \rightarrow \infty} u_n \neq 0$$

then the series  $\sum u_n$  is

- (i) convergent  
 (ii) divergent  
 (iii) not convergent  
 (iv) oscillatory

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(b) The  $p$ -series  $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$  to  $\infty$  is divergent for

- (i)  $p = 1$                       (ii)  $p > 1$   
 (iii)  $p < 1$                     (iv)  $p \leq 1$

(c) The series  $a - a + a - a + a - \dots$  to  $\infty$  is

- (i) convergent  
 (ii) divergent  
 (iii) oscillatory  
 (iv) None of the above

(d) The Laplace transform of a signal  $y(t)$  is

$$y(s) = \frac{1}{s(s-1)}$$

then its final value is

- (i) -1  
 (ii) 0  
 (iii) 1  
 (iv) unbounded

(e) The Laplace transform of the function  $f(t) = t$ , starting at  $t = a$  is

- (i)  $\frac{1}{(s+a)^2}$                       (ii)  $\frac{e^{-as}}{(s+a)^2}$   
 (iii)  $\frac{e^{-as}}{s^2}$                               (iv)  $\frac{a}{s^2}$

(f) If from the function  $f(t)$  one forms the function  $\psi(t) = f(t) + f(-t)$ , then  $\psi(t)$  is

- (i) even
- (ii) odd
- (iii) neither even nor odd
- (iv) both even and odd

(g) The triple integral  $\iiint_T dx dy dz$  gives

- (i) volume of region  $T$
- (ii) surface area of region  $T$
- (iii) area of region  $T$
- (iv) density of region  $T$

(h) The double integral

$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$$

is

- (i) 0
- (ii)  $\pi$
- (iii)  $\pi/2$
- (iv) 2

(i)  $\nabla \times (\nabla \times \vec{A})$ , where  $\vec{A}$  is a vector, is equal to

(i)  $\vec{A} \times \nabla \times \vec{A} - \nabla^2 \vec{A}$

(ii)  $\nabla^2 \vec{A} + \nabla(\nabla \cdot \vec{A})$

(iii)  $\nabla^2 \vec{A} + \nabla \times \vec{A}$

(iv)  $\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(j) If  $(\vec{A} \times \vec{B}) \times \vec{C} = \vec{A} \times (\vec{B} \times \vec{C})$ , then

(i)  $\vec{A}, \vec{B}$  are collinear

(ii)  $\vec{A}, \vec{B}$  are perpendicular

(iii)  $\vec{A}, \vec{C}$  are collinear

(iv)  $\vec{A}, \vec{C}$  are perpendicular

2. (a) Test for convergence the series whose  $n$ th term is  $n^{\log x}$ .

(b) Test the series for convergence

$$1 + \frac{3}{7}x + \frac{3 \cdot 6}{7 \cdot 10}x^2 + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13}x^3$$

$$+ \frac{3 \cdot 6 \cdot 9 \cdot 12}{7 \cdot 10 \cdot 13 \cdot 16}x^4 + \dots \text{ to } \infty$$

3. (a) Define absolutely and conditionally convergent series. Show that the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots \text{ to } \infty$$

is convergent but not absolutely convergent.

- (b) Find the Fourier series expansion of the periodic function of period  $2\pi$

$$f(x) = x^2, \quad -\pi \leq x \leq \pi$$

Hence find the sum of the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots \quad 14$$

4. (a) Find the Fourier series for  $f(x)$ , where

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

- (b) Find the Fourier half-range even expansion of the function

$$f(x) = -\frac{x}{e} + 1, \quad 0 \leq x \leq e \quad 14$$

5. (a) Find the Laplace transform of—

(i)  $t^2 \cos at$ ;

(ii)  $e^{-4t} \frac{\sin 3t}{t}$ .

- (b) Find the Laplace transform of  $\frac{1 - \cos t}{t^2}$ . 14

6. (a) Show that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$$

- (b) Evaluate  $\iint r^4 \cos^3 \theta \, dr \, d\theta$  over the interior of the circle  $v = 2a \cos \theta$ . 14

7. (a) Evaluate  $\int_1^e \int_1^{\log y} \int_1^x \log z \, dx \, dy \, dz$  akubihar.com

- (b) Find the length of the arc of the curve  $y = e^x$  from the point  $(0, 1)$  to  $(1, e)$ . 14

8. (a) Find the directional derivatives of  $\phi = xy^2 + yz^2$  at the point  $(2, -1, 1)$  in the direction of the vector  $\hat{i} + 2\hat{j} + 2\hat{k}$ .

- (b) Find the magnitude of the velocity and acceleration of a particle which moves along the curve  $x = 2 \sin 3t$ ,  $y = 2 \cos 3t$ ,  $z = 8t$  at any time  $t > 0$ . Find unit tangent vector to the curve. 14

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9. (a) If  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ , then prove that

$$\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2}$$

(b) Evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where

$$\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$$

and  $C$  is the rectangle in the  $xy$ -plane  
bounded by  $y=0$ ,  $x=a$ ,  $y=b$ ,  $x=0$ . 14

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