MUZAFFARPUR INSTITUTE OF TECHNOLOGY, MUZAFFARPUR, BIHAR – 842003

(Under the department of Science & Technology, Patna, Bihar)

B.Tech 3rd Semester Mid-Term Examination, 2018
FLUID MECHANICS

Time: 2 hours   Full Marks: 20
                                                                              Subject Code: 011307

1. **a) Enlist the different types of fluid flow. Define any two with example.**

Solution:

TYPES OF FLUID FLOW

The fluid flow is classified as :
(i) Steady and unsteady flows ;
(ii) Uniform and non-uniform flows ;
(iii) Laminar and turbulent flows ;
(iv) Compressible and incompressible flows ;
(v) Rotational and irrotational flows ; and
(vi) One, two and three-dimensional flows.

Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \ \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \ \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \ \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.,* length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity
           ∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow
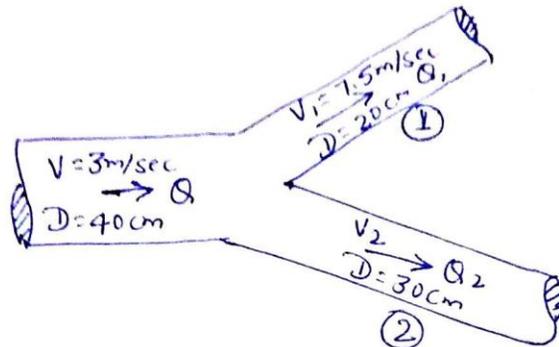
$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

b) A 40cm diameter pipe carrying water branches into two pipes of 20cm and 30cm diameters. The average velocity in the 40cm diameter pipe is 3m/s,

    i.   **Compute the discharge in this 40cm diameter pipe.**

    ii.   **If average velocity in 20cm diameter pipe is 7.5m/s, determine the velocity in 30cm diameter pipe.**

Soln:-   We have,

① Discharge (Q) in 40cm dia pipe

$$Q = A \cdot v = \left(\frac{\pi}{4} D^2\right) V = \left(\frac{\pi}{4} * 0.4\right) (3) = 0.377 \, m^3/sec$$

$$\Rightarrow \quad \boxed{Q = 0.377 \, m^3/sec} \quad A_2$$

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② Velocity in 30cm dia. pipe (V_2);

$$Q = Q_1 + Q_2 \Rightarrow Q = A_1 V_1 + A_2 V_2$$

$$\Rightarrow 0.377 = \left(\frac{\pi}{4} * 0.2^2\right) 7.5 + \left(\frac{\pi}{4} * 0.3^2\right) V_2$$

$$\Rightarrow \quad V_2 = 2m/sec.$$

2. **a) Write short notes on:**

    i.   **Total Energy Line (TEL)**

Total Energy Line.   It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).
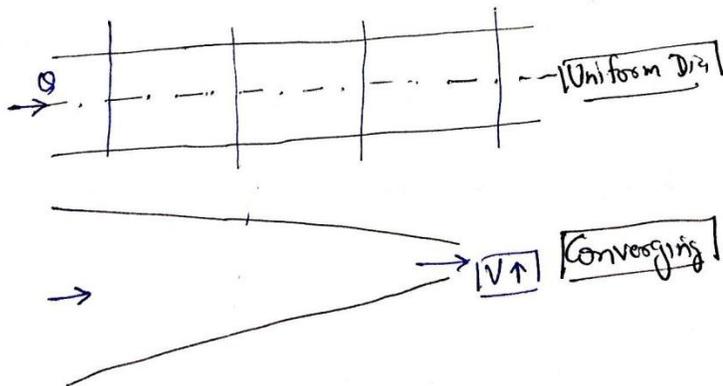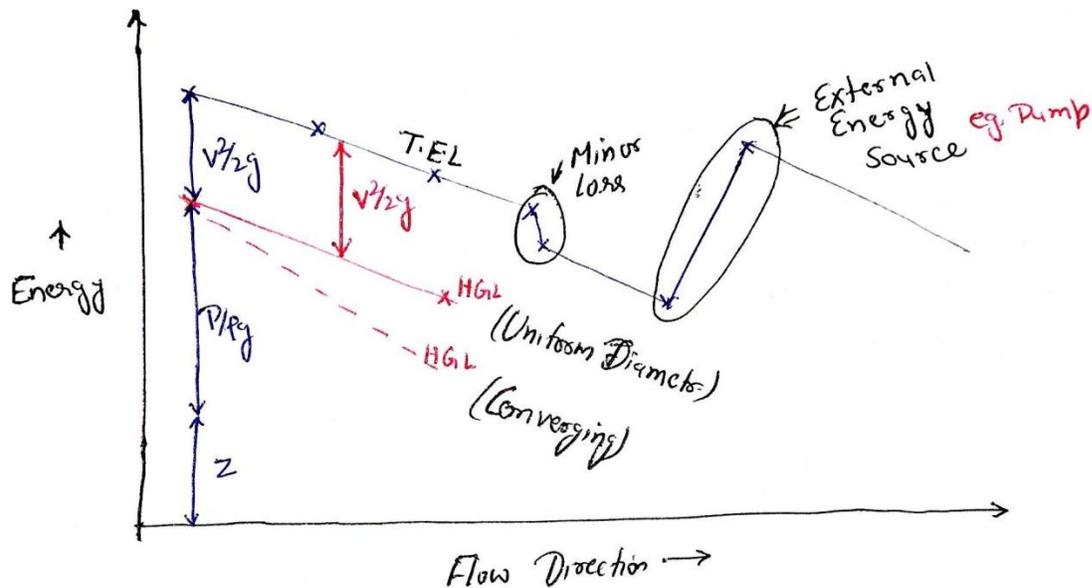
    ii.   Hydraulic Gradient Line (HGL)

Hydraulic Gradient Line.   It is defined as the line which gives the sum of pressure head $\left(\dfrac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

b) Find the head loss due to friction in a pipe of diameter 250mm length 60m, through which water is flowing at a velocity of 3m/s using

    i.   **Darcy formula**

    ii.   **Chezy`s formula for which C=55.**

            Take kinematic viscosity for water as 0.01 stoke

Soln:— We have,

$$D = 250mm,$$
$$L = 60m,$$
$$V = 3m/sec$$

① Darcy formula:

$$h_f = \frac{f \cdot L V^2}{2gD}$$

f = friction factor depends on Reynold's No (R_e)

$$R_e = \frac{\rho V D}{\mu} = \frac{V D}{\nu} = \frac{3 * 0.25}{0.01 * 10^{-4}} = 7.5 * 10^5.$$

for Pipe flow, $R_e = 7.5 * 10^5 > 4000$

$$\Rightarrow \text{Turbulent flow}$$

Thus, $f = \frac{0.316}{R_e^{1/4}} = \frac{0.316}{(7.5 \times 10^5)^{1/4}} = 0.0107$

Thus, head lost, $h_f = \frac{f \cdot L V^2}{2gD}$

$$= \frac{0.0107 * 60 * 3^2}{2 * 9.81 * 0.25} = \underline{1.18m}$$

$$\therefore \boxed{h_f = 1.18m} \quad A_2$$

② Chezy's formula:

$$V = C\sqrt{R \cdot S}$$

whose, $R = A/p = \frac{(\pi D^2/4)}{\pi D} = D/4 = \left(\frac{0.25}{4}\right)$

$$S = \frac{h_f}{L} =$$

$$\Rightarrow 3 = 55 \sqrt{\left(\frac{0.25}{4}\right)\left(\frac{h_f}{60}\right)}$$

$$\Rightarrow \boxed{h_f = 2.86m} \quad A_2$$

3. **a) Write short notes on:**

   i. **Streamline**   ii. **Pathline**   iii. **Streakline**

Streamline

- A streamline is an imaginary line drawn in a flow field such that a tangent drawn at any point on this line represents the direction of velocity vector at that point.
- There is no velocity component normal to stream lines.

Pathline

- A pathline is a curve traced by a single fluid particle during its motion.

Streakline

- When a dye is injected in a liquid or smoke in a gas so as to trace the subsequent motion of liquid particles passing a fixed point, the path followed by the dye or smoke is called the streakline.
- It is the locus of various fluid particles passing through a fixed point.
- In a steady flow streamline, pathline & streak lines are same.

b) The two scalar components of velocity field are given for two flow system. Find the third component of the velocity. For case:

$$u = x^3 + 2y^2 + z^3 \text{ and } v = -x^2 y - yz - xy$$

Solution: Velocity field should satisfy the continuity equation. For a three dimensional flow the equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

The partial derivatives of u and v are

$$\frac{\partial u}{\partial x} = 3x^2, \text{ and } \frac{\partial v}{\partial y} = -x^2 - z - x$$

Substituting these into the continuity equation, we get

$$\frac{\partial w}{\partial z} = x + z - 2x^2$$

Integrating we have,

Answer:

$$\boxed{w = xz + \frac{1}{2}z^2 - 2x^2 z + f(x,y)}$$

4. a) List types of fluid wit example.

Answer:

Types of Fluids. The fluids may be classified into the following five types :
1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

Types of fluids.

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.
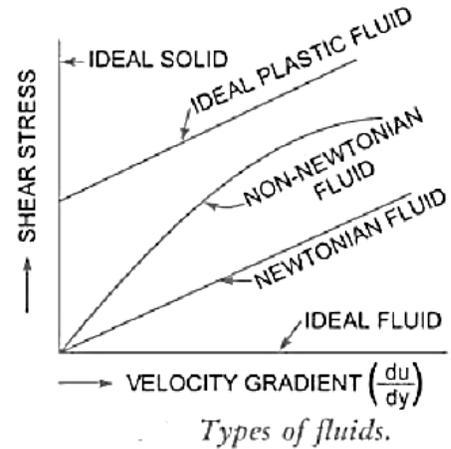
4. b)

Problem   *A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.*

Solution. Given :

Distance between plates, $dy = .025$ mm

$$= .025 \times 10^{-3} \text{ m}$$

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \dfrac{N}{m^2}$.

This is the value of shear stress *i.e.,* τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \dfrac{du}{dy}$.

where du = Change of velocity = $u - 0 = u = 0.60$ m/s

dy = Change of distance = $.025 \times 10^{-3}$ m

τ = Force per unit area = $2.0 \dfrac{N}{m^2}$

$\therefore$ $2.0 = \mu \dfrac{0.60}{.025 \times 10^{-3}}$ $\therefore$ $\mu = \dfrac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \dfrac{Ns}{m^2}$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise} = \textbf{8.33} \times \textbf{10}^{-4} \textbf{ poise. Ans.}$$
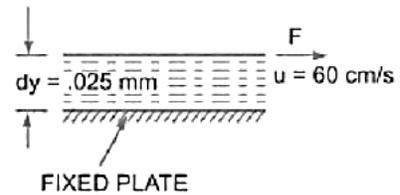
5. **Define metacenter and metacentric height. Derive expression to find metacentric height using analytical method for floating body.**

Answer:

META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (a). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.
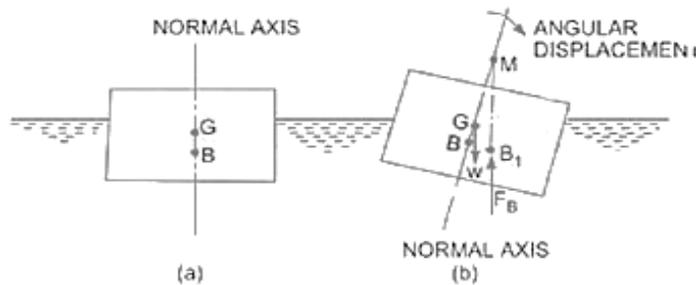

Fig. 4.5   Meta-centre

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (b). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (b). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

META-CENTRIC HEIGHT

The distance MG, i.e., the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

► ## ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 (a) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at G and B. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig. 4.6 (b). The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at M. Hence M is the meta-centre and GM is meta-centric height.
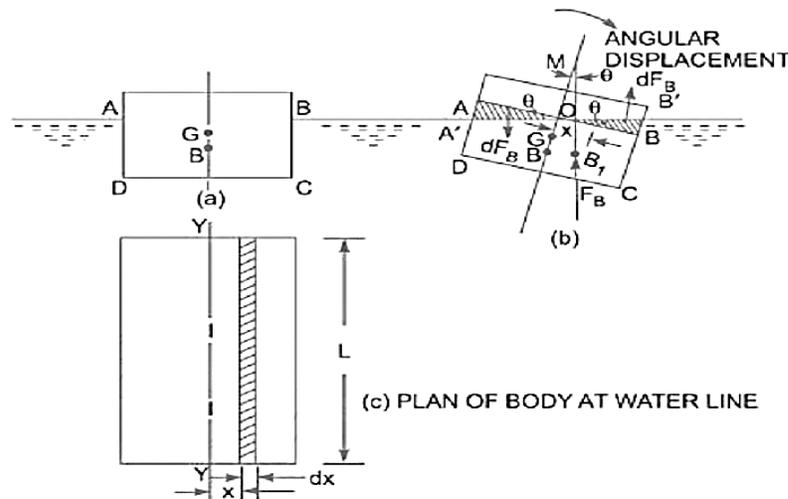

Fig.   Meta-centre height of floating body.

The angular displacement of the body in the clockwise direction causes the wedge-shaped prism *BOB'* on the right of the axis to go inside the water while the identical wedge-shaped prism represented by *AOA'* emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the *C.G.* of the prism *BOB'* while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of *AOA'*. The couple due to these buoyant forces dF_B tends to rotate the ship in the counterclockwise direction. Also the moment caused by the displacement of the centre of buoyancy from *B* to B_1 is also in the counterclockwise direction. Thus these two couples must be equal.

Couple Due to Wedges. Consider towards the right of the axis a small strip of thickness dx at a distance x from O as shown in Fig.　(b). The height of strip $x \times \angle BOB' = x \times \theta$.

$$\{ \because \angle BOB' = \angle AOA' = BMB_1' = \theta \}$$

∴　Area of strip 　 = Height × Thickness = $x \times \theta \times dx$

If L is the length of the floating body, then

Volume of strip 　　 = Area × L

　　　　　　　　 = $x \times \theta \times L \times dx$

∴　Weight of strip = $\rho g \times$ Volume = $\rho g x\, \theta L\, dx$

Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of strip will be $\rho g x \theta\, L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple 　　　　 = Weight of each strip × Distance between these two weights

　　　　　　　　　　　　 = $\rho g x\, \theta L\, dx\, [x + x]$

　　　　　　　　　　　　 = $\rho g x\, \theta L\, dx \times 2x = 2\rho g x^2\, \theta L\, dx$

∴　Moment of the couple for the whole wedge

　　　　　　　　　　　 = $\int 2\rho g x^2\, \theta L\, dx$ 　　　　　　　 ...　1

Moment of couple due to shifting of centre of buoyancy from B to B_1

　　　　　　　　 = $F_B \times BB_1$

　　　　　　　　 = $F_B \times BM \times \theta$ 　　　　 $\{ \because BB_1 = BM \times \theta$ if θ is very small$\}$

　　　　　　　　 = $W \times BM \times \theta$ 　　　　　　 $\{ \because F_B = W \}$ ...　2

But these two couples are the same. Hence equating equations 　1 and 　2 , we get

　　　　 $W \times BM \times \theta = \int 2\rho g x^2\, \theta\, Ldx$

　　　　 $W \times BM \times \theta = 2\rho g\theta \int x^2 Ldx$

　　　　 $W \times BM = 2\rho g \int x^2 Ldx$

Now Ldx = Elemental area on the water line shown in Fig. 　(c) and = dA

∴　　　　　　 $W \times BM = 2\rho g \int x^2 dA.$

But from Fig. 　(c) it is clear that $2 \int x^2 \, dA$ is the second moment of area of the plan of the body at water surface about the axis *Y-Y*. Therefore

　　　　　　 $W \times BM = \rho g I$ 　　　　　 $\{$where $I = 2 \int x^2 \, dA \}$

∴ 　　　　　　　 $BM = \dfrac{\rho g I}{W}$

But 　　　　　　 W = Weight of the body

　　　　　　　 = Weight of the fluid displaced by the body

　　　　　　　 = $\rho g \times$ Volume of the fluid displaced by the body

　　　　　　　 = $\rho g \times$ Volume of the body sub-merged in water

　　　　　　　 = $\rho g \times \forall$

∴ 　　　　　 $BM = \dfrac{\rho g \times I}{\rho g \times \forall} = \dfrac{I}{\forall}$ 　　　　　 ...　3

　　　　 $GM = BM - BG = \dfrac{I}{\forall} - BG$

∴ Meta-centric height 　 = $GM = \dfrac{I}{\forall} - BG.$ 　　　　 ...　4

6. a) Write down Bernoulli's principle and equation.

In fluid dynamics, Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy. Although Bernoulli deduced that pressure decreases when the flow speed increases. Bernoulli's principle can be derived from the principle of conservation of energy. This states that, in a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline. This requires that the sum of kinetic energy, energy and internal energy remains constant.

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

∴ $$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or $$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

υ $$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad \text{........ } 1$$

Equation **1** is a Bernoulli's equation in which

$$\frac{p}{\rho g} = \text{pressure energy per unit weight of fluid or pressure head.}$$

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

6.b)

Problem Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Solution. Given :

Diameter of pipe	= 5 cm = 0.5 m
Pressure,	$p = 29.43$ N/cm² $= 29.43 \times 10^4$ N/m²
Velocity,	$v = 2.0$ m/s
Datum head,	$z = 5$ m
Total head	= pressure head + kinetic head + datum head

Pressure head $$= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m} \qquad \left\{ \rho \text{ for water} = 1000 \ \frac{\text{kg}}{\text{m}^3} \right\}$$

Kinetic head $$= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$$

∴ Total head $$= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = 35.204 \text{ m. Ans.}$$