Mechanical Engg. dept.

3rd sem, Fluid Mechanics

sol^n of mid term question paper.

(i) velocity Potential:- It is defined as a scalar function of space and time such that its -ve derivative with respect to any direction gives the velocity of fluid in that direction. It is denoted as 'ϕ'

For steady flow, $\phi = f(x, y, z)$ i.e. $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$, $w = -\frac{\partial \phi}{\partial z}$

where u, v, w are the component of velocity in, x, y and z dir^n respectively.

* stream function:- It is defined as scalar function of space & time such that its partial derivative w.r.t. any dir^n gives the velocity components at right angles to that dir^n. It is denoted by 'ψ'

i.e. $\frac{\partial \psi}{\partial x} = v$, and $\frac{\partial \psi}{\partial y} = -u$

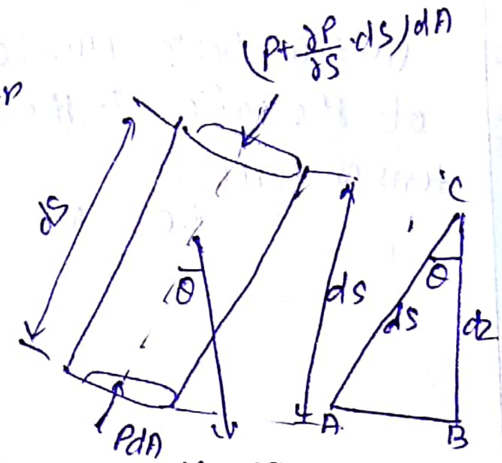
Here, u, v are the component of velocity in, x and y dir^n respectively.

Q.2:- Statement:- "In a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline."

Assumption:- The following assumptions to be made in derivation of Bernoulli's eq^n:- (i) The fluid is ideal i.e. $\mu = 0$, (ii) The flow is steady (iii) The flow is incompressible (iv) The flow is irrotational.

Derivation:- Consider a stram-line in which flow is taking place in s-dir^n as shown in fig. Consider a cylindrical element of cross-section 'dA' and length 'ds'

The resultant force on the fluid element in s-dir^n



$PdA - (P + \frac{\partial P}{\partial s} ds)dA - \rho g dA \cdot ds \cdot \cos\theta = \rho dA ds \cdot a_s$

$a_s = \frac{dv}{dt} = \frac{\partial u}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial u}{\partial t} = v\frac{\partial u}{\partial s} + \frac{\partial u}{\partial t}$

For steady flow. $\frac{\partial u}{\partial t} = 0$

$a_s = v\frac{\partial u}{\partial s}$

$\omega = mg = \rho \cdot dA \cdot ds \cdot g$

In ΔABC

$\cos\theta = \frac{dz}{ds}$

$ds\cos\theta = dz$

or, $\cancel{PdA} - \cancel{PdA} - \frac{\partial P}{\partial s} \cdot ds \cdot dA - \rho g dA\, dz = \rho dA ds \cdot v\frac{\partial v}{\partial s}$

$\Rightarrow -dA \cdot ds \left(\frac{\partial P}{\partial s} + \rho g \frac{dz}{\partial s} + \rho v \frac{\partial v}{\partial s} \right) = 0$

or, $\frac{\partial P}{\partial s} + \rho g \frac{dz}{\partial s} + \rho v \frac{\partial u}{\partial s} = 0$

$\Rightarrow \boxed{\frac{dP}{\rho} + udu + gdz = 0}$

Now integrating the eqn.

$$\int \frac{\partial p}{\rho} + \int g\,dz + \int u\,du = const.$$

If flow is incompressible. i.e. ρ is const.

$$\frac{p}{\rho} + gz + \frac{u^2}{2} = Const.$$

i.e. $$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

where $\frac{P_1}{\rho g}$ = Pressure head, $\frac{u^2}{2g}$ = Kinetic head, and z = Potential head

Q3. Reynold's No:- It is defined as the ratio of Inertia force of a flowing fluid and the viscous force of the fluid.

i.e. $$Re = \frac{\text{Inertia force } (F_i)}{\text{viscous force } (F_v)} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

(ii) Froude's Number (Fe):- It is defined as the square root of the ratio of Inertia force of a flowing fluid to the gravity force.

i.e. $$Fe = \sqrt{\frac{\text{inertia Force } (F_i)}{\text{gravity Force } (F_g)}} = \frac{V}{\sqrt{Lg}}$$

(iii) Mach's Number (M):- It is defined as the square root of the ratio of the Inertia force of a fluid to the elastic force.

i.e. $$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$$

(iv) Weber's Number (We) = It is defined as the square root of the ratio of the Inertia force of a flowing fluid to the Surface tension force.

i.e. $$We = \sqrt{\frac{F_i}{F_s}} = \frac{V}{\sqrt{\sigma/\rho L}}$$

Skum

Q.4. Given

$H = 900mm = 0.9m$

$Q = 300 \text{ lit/sec} = 0.3 m^3/s$

$C_d = 0.62$

we know that

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$0.3 = \frac{2}{3} \times 0.62 \times L \times \sqrt{2 \times 9.81} \times (0.9)^{3/2}$$

$$\therefore L = 0.192 m = 192 mm$$

Q.5. Given $u = 2y - y^2$

$$\frac{du}{dy} = 2 - 2y$$

$\mu = 0.4$ Poise $= 0.04 N \cdot s/m^2$

(i) $\left(\frac{du}{dy}\right)_{y=0} = 2$

$$\tau = \mu \cdot \frac{du}{dy} = 0.04 \left(\frac{du}{dy}\right)_{y=0} = 0.04 \times 2 = 0.08 N/m^2$$

(ii) $\left(\frac{du}{dy}\right)_{y=0.05} = 2 - 2 \times (0.05) = 2 - 0.1 = 1.9$

$$\tau = \mu \cdot \frac{du}{dy} = 0.04 \left(\frac{du}{dy}\right)_{0.05} = 0.04 \times 1.9$$

$$= 0.076 N/m^2$$

Q.6. Given $d_1 = 100mm = 10cm$

$a_1 = \frac{\pi}{4}(d_1)^2 = 78.539 cm^2$

contraction ratio $(C_c) = 0.5$

Dia. of throat $(d_2) = 0.5 \times d_1 = 0.5 \times 10 = 5 cm$

$a_2 = \frac{\pi}{4}(5)^2 = 19.635 cm^2$

Head of water (for no flow) $= \frac{P_1}{\rho g} = 3m$ (gauge)

absolute pressure head $= 3 + 10.3 = 13.3 m$

Total pressure head $= \frac{P_2}{\rho g} = 2m$ of water (absolute)

Diff. in Pressure head (absolute) $= \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$

$= 13.3 - 2$

$= 11.3 = 1130\,cm$ of water

rate of flow

$Q_1 = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$= 0.97 \times \frac{78.539 \times 19.635}{\sqrt{(78.539)^2 - (19.635)^2}} \sqrt{2 \times 9.81 \times 1130}$

$= 29289.1\ cm^3/sec$

$= 29.2891\ lit/sec$