2018-19

5th Sem, B.Tech in Electrical Engg.

EMRT

1. Select one answer out of multiple choice question

(i) A wedge is described by $z=0$, $30^\circ < \phi < 60^\circ$, $\rho = 2$ which of the following is incorrect

(a) The wedge lies in the xy-plane

Correct (b) It is infinitely long

(c) A unit normal to the wedge is $\pm \hat{a}_z$

(d) The wedge includes neither the x-axis nor the y-axis

(ii) which of the following is zero?

(a) grad div

(b) div grad

(c) Curl grad

(d) Curl curl

(iii) Point charge 30 nC, -20 nC, and 10 nC are located at (-1, 0, 2), (0, 0, 0) and (1, 5, -1), respectively. The total flux leaving a cube of side 6m centered at the origin is

(a) 20 nC

(b) -20 nC

(c) 10 nC

(d) -10 nC

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(iv) The relaxation time of mica ($\sigma = 10^{-15}$ S/m, $\varepsilon_r = 6$) is

(a) 5×10^{-10} s

(b) 10^{-6} s

(c) 5 hr

✓(d) 15 hr

(v) In cylindrical coordinates, the equation

$$\frac{\partial^2 \psi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \psi}{\partial \phi} + \frac{\partial^2 \psi}{\partial z^2} + 10 = 0$$ is called

(a) Maxwell's equation

✓(b) Poisson's equation

(c) Laplace's equation

(d) Helmholtz's equation

(vi) One of the following is not a source of magnetostatic fields:

(a) A dc current in a wire

(b) A permanent magnet

✓(c) An accelerated charge

(d) A charged disk rotating at uniform speed

(vii) If $\bar{E}_s = 10 e^{j4x} \hat{a}_y$, which of these is not a corrected representation of $\bar{E}$?

(a) $Re(\bar{E}_s e^{j\omega t})$

(b) $Re(\bar{E}_s e^{-j\omega t})$

✓(c) $10 \cos(\omega t + j4x) \hat{a}_y$

(d) $10 \sin(\omega t + 4x) \hat{a}_y$

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(iii) In a certain medium, $E = 10\cos(10^8 t - 3y)\,\hat{a}_x$ V/m. What type of medium is it?

✓(a) Lossless dielectric

(b) Perfect conductor

(c) lossy dielectric

(d) Free space

(iv) The concept of displacement current was a major contribution attributed to

(a) faraday

(b) Lorentz

(c) ~~III~~ Lenz

✓(d) maxwell

(x) The value of α for lossless medium is

✓(a) 0

(b) 0.1

(c) -0.1

(d) None of the above

Q.2. State and derive Uniqueness Theorem

If a solution to Laplace's equation can be found that satisfies the boundary conditions, then the solution is unique.

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The theorem is proved by contradiction. We assume that there are two solutions V_1 and V_2 of Laplace's equation, both of which satisfy the prescribed boundary conditions. Thus

$$\nabla^2 V_1 = 0 , \quad \nabla^2 V_2 = 0 \qquad \text{—— (1)}$$

$$V_1 = V_2 \quad \text{on the boundary}$$

let us take their difference

$$V_d = V_2 - V_1 \qquad \text{—— (2)}$$

$$\nabla^2 V_d = \nabla^2 V_2 - \nabla^2 V_1 = 0 \qquad \text{—— (3)}$$

$$V_d = 0 \quad \text{on the boundary}$$

From divergence theorem, $\int_v (\nabla \cdot \bar{A})\, dv = \oint_s \bar{A} \cdot d\bar{s}$ —— (4)

we let $\bar{A} = V_d \nabla V_d$,

As we know that $\nabla \cdot \bar{A} = \nabla \cdot (V_d \nabla V_d) = V_d \nabla^2 V_d + \nabla V_d \cdot \nabla V_d$

But $\nabla^2 V_d = 0$ from (3)

$$\nabla \cdot \bar{A} = \nabla V_d \cdot \nabla V_d \qquad \text{—— (5)}$$

$$\int \nabla V_d \cdot \nabla V_d \, dv = \oint_s V_d \nabla V_d \cdot d\bar{s}$$

$$\int |\nabla V_d|^2 dv = 0$$

$$\nabla V_d = 0$$

$$V_d = V_2 - V_1 = \text{Constant everywhere in } v$$

Hence $V_d = 0$ or $V_1 = V_2$ everywhere, showing that V_1 and V_2 cannot be different solutions of the same problem.

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1.3. A lossy dielectric has an intrinsic impedance of $200\angle 30^\circ\ \Omega$ at a particular radian frequency ω. If, at that frequency, the plane wave propagating through the dielectric has the magnetic field component

$$\bar{H} = 10\, e^{-\alpha x} \cos\left(\omega t - \frac{1}{2}x\right) \hat{a}_y \ \text{A/m}$$

Find $\bar{E}$ and α.

Solution The given wave travels along $\hat{a}_x$ so that

$$\hat{a}_K = \hat{a}_x \ ; \ \hat{a}_H = \hat{a}_y \text{, so}$$

$$-\hat{a}_E = \hat{a}_K \times \hat{a}_H = \hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{a}_E = \hat{a}_z$$

Also, $H_o = 10$, So

$$\frac{E_o}{H_o} = \eta = 200\angle 30^\circ = 200\, e^{j\pi/6}$$

$$E_o = 2000\, e^{j\pi/6}$$

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$$\bar{E} = \mathrm{Re}\left[2000\, e^{j\pi/6} e^{-\gamma x} e^{j\omega t} \hat{a}_E\right]$$

$$\bar{E} = -2\, e^{-\alpha x} \cos\left(\omega t - \frac{x}{2} + \pi/6\right) \hat{a}_z \ \text{kV/m}$$

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$\beta = \frac{1}{2}$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)}$$

$$\frac{\alpha}{\beta} = \frac{1}{\sqrt{3}} \quad ; \quad \frac{\sigma}{\omega\varepsilon} = \tan 2\theta_\eta = \tan 60^\circ = \sqrt{3}$$

$$\alpha = 0.2887 \ Np/m$$

Q. 4. Two homogeneous isotropic dielectrics meet on plane $z=0$. For $z>0$, $\varepsilon_{r_1} = 4$ and $z<0$, $\varepsilon_{r_2} = 3$. A uniform electric field $\bar{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m exists for $z \geq 0$. Find $\bar{E}_2$ for $z \leq 0$

Sol.

$$\bar{E}_{1n} = \bar{E}_1 \cdot \hat{a}_n = \bar{E}_1 \cdot \hat{a}_z = 3$$

$$\bar{E}_{1n} = 3\hat{a}_z$$

$$\bar{E}_{2n} = (\bar{E}_2 \cdot \hat{a}_z)\hat{a}_z$$

$$\bar{E} = \bar{E}_n + \bar{E}_t$$

$$\bar{E}_{1t} = \bar{E}_1 - \bar{E}_{1n} = 5\hat{a}_x - 2\hat{a}_y$$

$$\bar{E}_{2t} = 5\hat{a}_x - 2\hat{a}_y$$

$$\bar{D}_{2n} = \bar{D}_{1n} \Rightarrow \varepsilon_{r_2}\bar{E}_{2n} = \varepsilon_{r_1}\bar{E}_{1n}$$

$$\bar{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\bar{E}_{1n} = \frac{4}{3}(3\hat{a}_z) = 4\hat{a}_z$$

$$\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}$$

$$= (5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z) \ kV/m$$

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