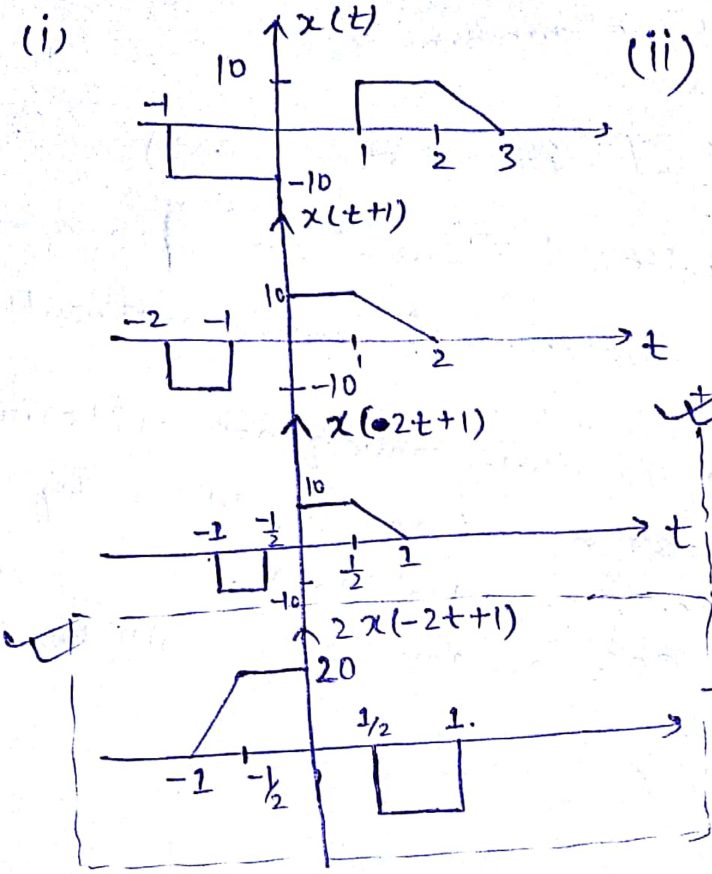
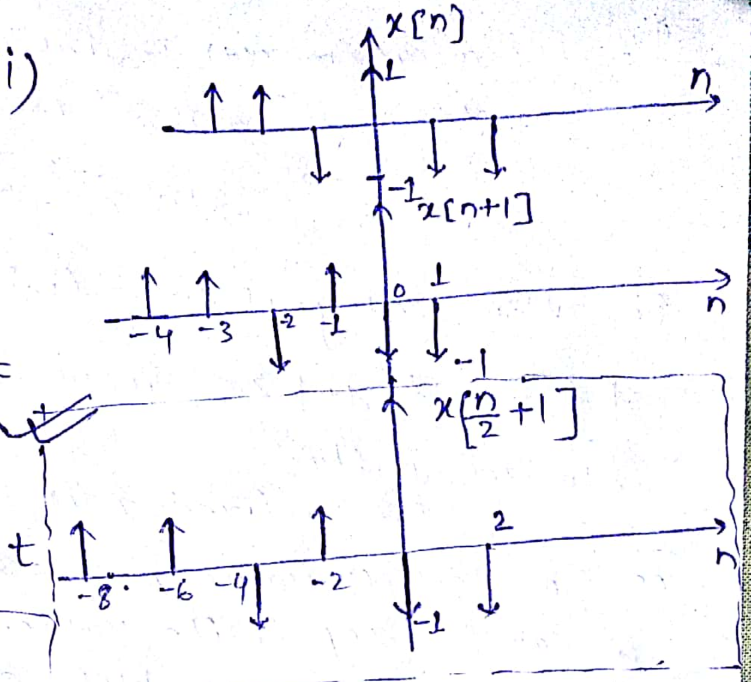
Q.1. (i)



(ii)



(iii)
$$I = \int_{-\infty}^{10} \left[t^2 . e^{\frac{t}{2}} \{\delta(-2t-1) + \delta(t-12)\}\right] dt$$
$$= \int_{-\infty}^{10} t^2 . e^{\frac{t}{2}} \delta(-2t-1)\,dt + \cancel{\int_{-\infty}^{10} t^2 . e^{\frac{t}{2}} \delta(t-12)\,dt}^{\,0}$$
$$= \left(t^2 . e^{t/2} . \tfrac{1}{2}\right)_{t=-\frac{1}{2}} + 0 \qquad \left(\because \delta(at) = \frac{1}{|a|}\delta(t)\right)$$
$$\boxed{I = \frac{1}{8} . e^{-\frac{1}{4}}}$$

(iv) Given ROC for $x_1[n] + x_2[n] \Rightarrow \frac{1}{3} < |z| < \frac{2}{3}$
then ROC for $x_1[n] - x_2[n]$ will also be $\frac{1}{3} < |z| < \frac{2}{3}$
as ROC remains the same for addition and subtraction in z-domain.

iv) $x_1[n] = \{0 \ \underset{\uparrow}{3} \ 6 \ 9\}$; $x_2[n] = \{1 \ \underset{\uparrow}{2} \ 3 \ 4\}$

$x_1[n] \circledast x_2[n] =$

	0	3	6	9
1	0	3	6	9
2	0	6	12	18
3	0	9	18	27
4	0	12	24	36

$\therefore x_1[n] \circledast x_2[n]$
$= \{0 \ 3 \ \underset{\uparrow}{12} \ 30 \ 48 \ 51 \ 36\}$

Q. 2(i)

a) $y[n] = sgn(x[n])$

* It is a non-linear system, as it does't follow the additivity property.

i.e. $y_1[n] = sgn(x_1[n])$

$y_2[n] = sgn(x_2[n])$

$\therefore$ $y_1[n] + y_2[n] = sgn(x_1[n]) + sgn(x_2[n])$ —— (I)

but $y_{1+2}[n] = sgn(x_1[n] + x_2[n])$ —— (II)

since (I) $\neq$ (II) this system is non-linear.

* It is a static system as o/p at any instant of time is depending on present input.

* Causal system.

$\boxed{\text{Non-Linear, static \& causal}}$

(b) $y[n] = x^2[n]$

* Non-linear as it doesn't follow the additivity property.
* static because o/p $x[n] \cdot x[n]$ at any instant is depending on present i/p $x[n]$.
* Causal system.

$\boxed{\text{Non-linear, static \& Causal.}}$

2)(ii) $x(t) = e^{-2t} u(t) \longleftrightarrow X(s)$

(a) $Y_1(s) = X(2s)$ then $y_1(t) = ?$

$\because$ $x(at) \xleftrightarrow{L.T} \frac{1}{|a|} \cdot X\left(\frac{s}{a}\right)$

or $|a| \cdot x(at) \xleftrightarrow{L.T} X\left(\frac{s}{a}\right)$

or, $X\left(\frac{s}{a}\right) \xleftrightarrow{I.L.T} |a| \cdot x(at)$

put $a = 1/2$. , $X(2s) \longleftrightarrow \frac{1}{2} \cdot x(t/2)$

$\therefore$ $\boxed{y_1(t) = \frac{1}{2} \cdot e^{-t} u(t)}$, $\left[\text{as } x(t) = e^{-2t} \cdot u(t)\right]$

(2) (ii) (b) $Y_2(s) = \frac{d}{ds} X(s)$

$\because -t \cdot x(t) \xleftrightarrow{L.T} \frac{d}{ds} X(s)$

$\therefore \frac{d}{ds} X(s) \xleftrightarrow{I.L.T} -t \cdot x(t)$

$\longleftrightarrow -t \cdot e^{-2t} \cdot u(t)$

4 $\therefore \boxed{y_2(t) = -t \cdot e^{-2t} u(t)}$

(C)

$Y_3(s) = s \cdot X(s).$

$\because \frac{d}{dt} x(t) \xleftrightarrow{L.T} s \cdot X(s)$

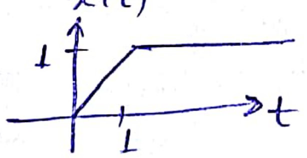
$\therefore y_3(t) = \frac{d}{dt} x(t)$

$= \frac{d}{dt} e^{-2t} \cdot u(t) = -2e^{-2t} \cdot u(t) + e^{-2t} \cdot \delta(t)$

$= \delta(t) - 2e^{-2t} \cdot u(t).$

4 $\boxed{\therefore y_3(t) = \delta(t) - 2e^{-2t} u(t)}$

(2) (iii)



$x(t) = t \cdot u(t) - t \cdot u(t-1)$

Taking Laplace transform,

$X(s) = \mathcal{L}\{ t \cdot u(t) - t \cdot u(t-1) \}$

$= \frac{1}{s^2} - \frac{e^{-s}}{s^2}$

4 $\boxed{X(s) = \frac{1}{s^2}(1 - e^{-s})}$

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(3.) (i) $X(s) = \frac{1}{s^2+5s+6}$

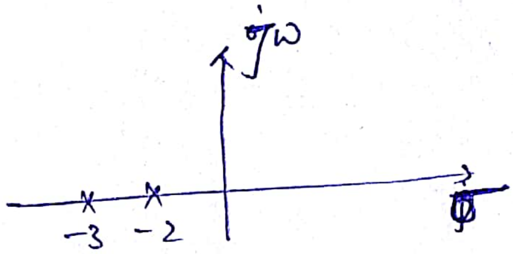
$$= \frac{1}{s^2+3s+2s+6} = \frac{1}{s(s+3)+2(s+3)}$$

$$= \frac{1}{(s+3)(s+2)}$$

$$\therefore X(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A\Big|_{s=-2} = \frac{1}{-2+3} = 1.$$

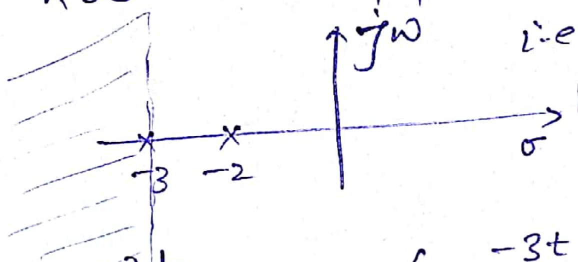
$$B\Big|_{s=-3} = \frac{1}{-3+2} = -1.$$



$$\therefore X(s) = \frac{1}{s+2} - \frac{1}{s+3}$$

Case-I. When ROC is $Re\{s\} < -3$.

i.e.

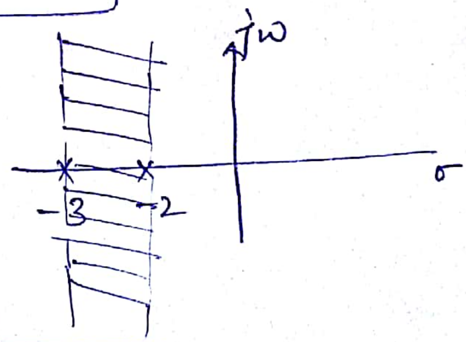


i.e. for both poles it is left handed signal.

$$\therefore x(t) = -e^{-2t}.u(-t) - (-e^{-3t}.u(-t))$$

$$\boxed{x(t) = e^{-2t}u(-t) + e^{-3t}u(-t)}$$

Case-II. When ROC is $-3 < Re\{s\} < -2$.



For pole -3, it is right handed signal but for pole -2, it is left handed signal.

$$\therefore \boxed{x(t) = -e^{-2t}u(-t) - e^{-3t}.u(t).}$$

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3.(iii)

$$x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u[n]$$

$$= \left(\frac{1}{3}\right)^{-n} \cdot u[-n-1] + \left(\frac{1}{3}\right)^{n} \cdot u[n] - \left(\frac{1}{2}\right)^{n} u[n]$$

let $x_1[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$

$$\therefore X_1(z) = \sum_{-1}^{-\infty} \left(\frac{1}{3}\right)^{-n} \cdot z^{-n}$$

$$= \sum_{-1}^{-\infty} \left(\frac{z}{3}\right)^{-n}$$

$$= \frac{z}{3} + \left(\frac{z}{3}\right)^2 + \left(z/3\right)^3 + \cdots$$

$$= \frac{z/3}{1 - z/3}, \quad |z/3| < 1$$

$$= \frac{z}{3-z}, \quad |z| < 3.$$

let $x_2[n] = \left(\frac{1}{3}\right)^{n} u[n]$.

$$\therefore X_2(z) = \frac{z}{z - 1/3}, \quad |z| > 1/3$$

(By property)

$\because \left[a^n u[n] \xleftrightarrow{Z.T} \frac{z}{z-a}, |z| > |a|\right]$

& let $x_3[n] = \left(\frac{1}{2}\right)^{n} \cdot u[n]$.

$$\therefore X_3(z) = \frac{z}{z - 1/2}, \quad |z| > 1/2.$$

Since Z-transforms follows the linearity,

$$\therefore X(z) = Z.T.\{x_1[n] + x_2[n] + x_3[n]\}$$

$$\boxed{X(z) = \frac{z}{3-z} + \frac{3z}{3z-1} - \frac{2z}{2z-1}},$$

& the common ROC will be,

For, $X_1(z)$, $|z| < 3$

$X_2(z)$, $|z| > 1/3$

$X_3(z)$, $|z| > 1/2$

$\therefore$ $\boxed{\text{Common ROC } \frac{1}{2} < |z| < 3.}$

3(ii) For a system $x(t)$, its Laplace transform is,

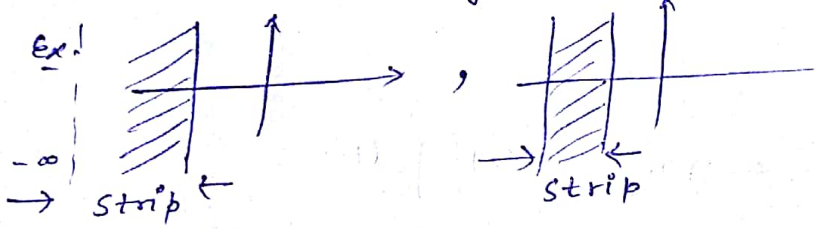
$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$$
$$= \int_{-\infty}^{\infty} x(t) \; e^{-\sigma t} \cdot e^{-j\omega t} dt \quad (\text{as } s = \sigma + j\omega)$$

For this integral to be finite $\left| \int_{-\infty}^{\infty} x(t) \, e^{-\sigma t} \cdot e^{-j\omega t} dt \right| < \infty$, or $\left| \int_{-\infty}^{\infty} x(t) \, e^{-\sigma t} dt \right| < \infty$, as the amplitude of complex exponential be always unity.

For a fixed $x(t)$, the whole convergence criteria depends on σ i.e. (ROC) value. & doesn't depend on 'ω'. It simply means ω can be any finite value. i.e. $-\infty < \omega < \infty$.

So if we plot ROC for any 'σ' it will be a strip parallel to vertical (y) axis.

Ex:



* For a sequence $x[n]$ the z-transform is defined as.

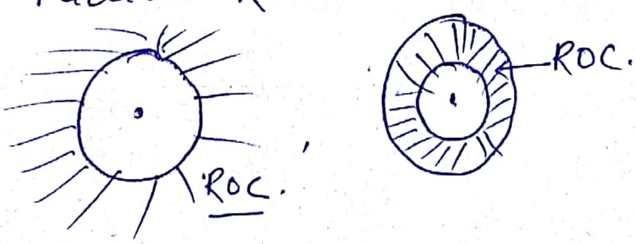
$$X(z) = \sum_{-\infty}^{\infty} x[n] \, z^{-n}$$
$$= \sum_{-\infty}^{\infty} x[n] \cdot r^{-n} \, e^{-j\omega n} \quad \text{as } (z = re^{j\omega} = r\angle\omega,$$ r is radius & ω is angle.

For this summation to be finite, $X(z) = \left| \sum_{-\infty}^{\infty} x[n] \cdot r^{-n} \cdot e^{-j\omega n} \right| < \infty$ or $\left| \sum_{-\infty}^{\infty} x[n] \cdot r^{-n} \right| < \infty$, as the amplitude of complex exponential be always unity.

So for a fixed $x[n]$, the whole criteria depends on value of r which is radius & doesn't depend on 'ω'. So it can be concluded that 'ω' can be any angle from 0 to 2π.

Hence if we plot ROC for z-transform it will be a circle of radius 'R'.

ex.



Condition for Stability:-

* In S-domain if ROC consists $\sigma = 0$ line, then system is stable otherwise unstable.
* In Z-domain if ROC consists $r = 1$ or unit radius circle then system is stable otherwise unstable.

Condition for causality:-

* In S-domain if ROC is right side of the rightmost pole, then system is causal other wise non-causal.
* In Z-domain if ROC is outside of the largest pole, then system is causal other-wise non-causal.