MUZAFFARPUR INSTITUTE OF TECHNOLOGY, MUZAFFARPUR, BIHAR – 842003

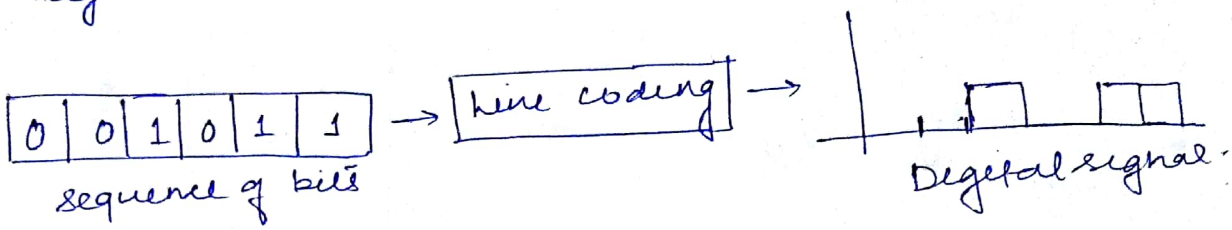
(Under the department of Science & Technology, Bihar, Patna)

B.Tech 7th Semester Mid-Term Examination, 2018

E-III (Digital Communication & Telecommunication Management)

Solution

1(A) <u>Line coding format</u> :- It is the process of converting digital data (i.e binary data, sequence of bits) into digital signal / electrical pulse. It is also called digital to digital conversion.



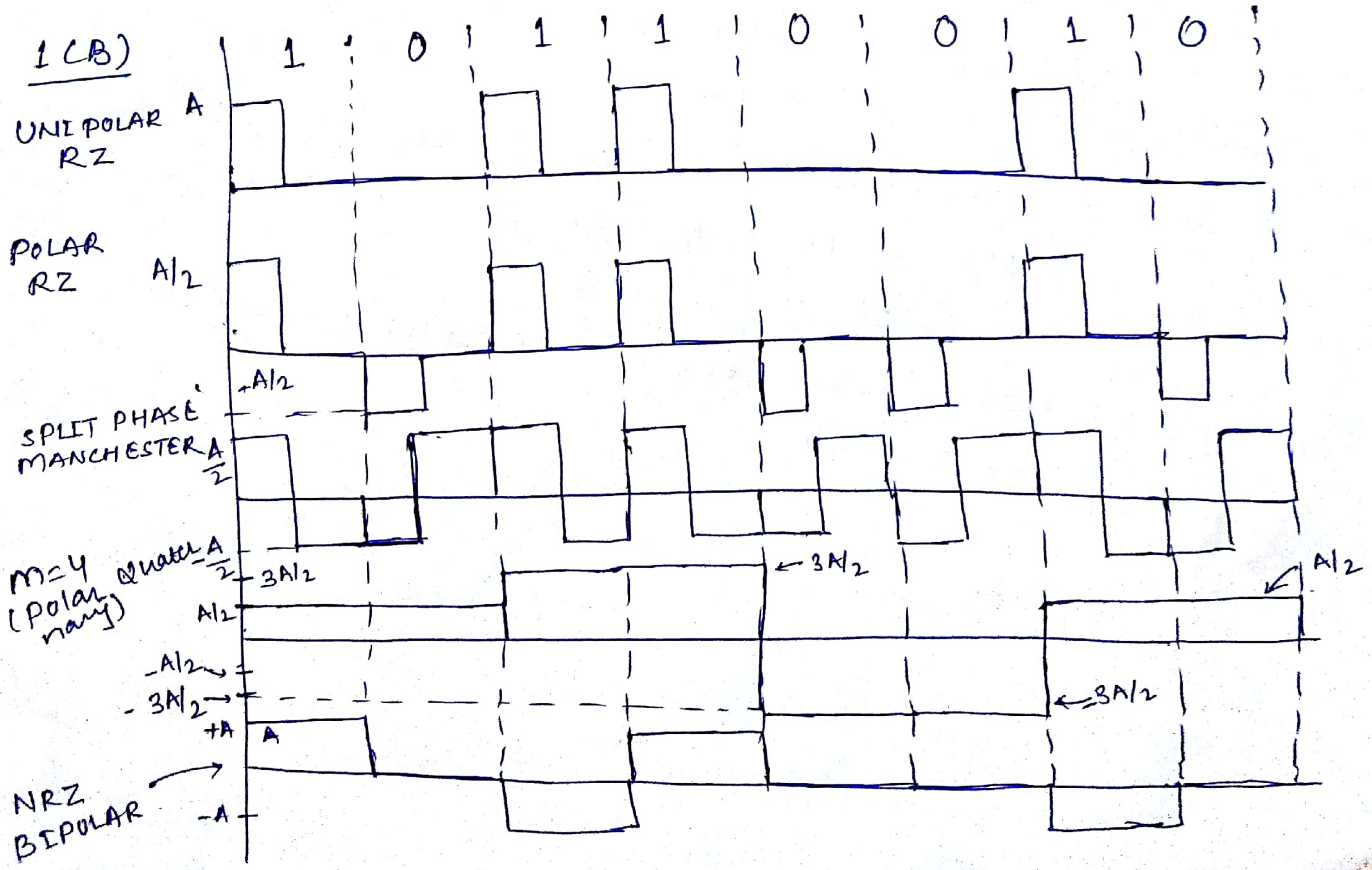
<u>Desirable properties of line coding</u>

1. Transmission bandwidth :- As small as possible.
2. power efficiency :- For a given bandwidth and specified detection error probability, the transmission mitted power for a line code should be ~~can~~ small as possible.
3. Error detection & correction probability :- It must be possible to detect & preferably correct detection errors
4. favourable power spectral density :- It is desirable to have zero power spectral density (PSD) at w=0 {i.e DC).
5. Cross talks between the channel must be minimized.

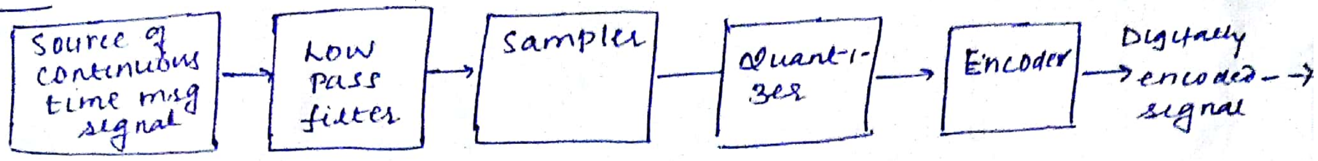
Comparison of various line codes

Parameters	Polar RZ	Polar NRZ	AMI NRZ	Manchester	Polar Quarternary NRZ
1. Transmission of DC component	yes	yes	NO	NO	possible
2. Signalling rate	$1/T_b$	$1/T_b$	$1/T_b$	$1/T_b$	$1/2T_b$
3. Noise immunity	Low	Low	high	high	high
4. Synchronizing capacity	poor	poor	very good	very good	poor
5. B.W Reqd.	$1/T_b$	$\frac{1}{2T_b}$	$\frac{1}{T_b}$	$\frac{1}{T_b}$	$\frac{1}{2T_b}$
6 Cross talks	High	High	Low	Low	Low

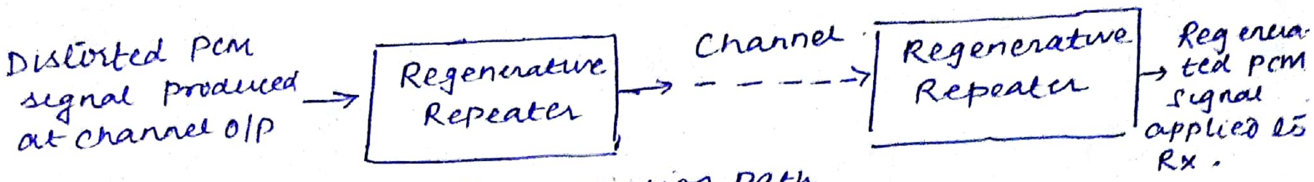
1 (B)



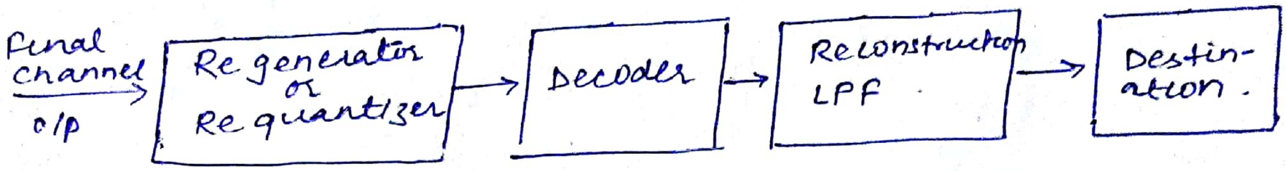
2 (A)



Transmitter.



Transmission path



Receiver

Working

1. The modulating signal m(t) is first passed through a sampler to convert the continuous time signal into into a discrete time signal. The quantiser converts the discrete in time and continuous in value signal into discrete in time and discrete in value signal (digital signal)

2. The encoder assigns N no. of bits to each sample and finally digitally encoded PCM signal is transmitted over the channel.

3. At the receiving end the signal is passed through a regenerator or re-quantiser which removes the additive noise present in the signal and generates fresh digital pulses at its output.

4. The decoder converts the digitally encoded signal into corresponding analog value and with the help of LPF, the original signal is reconstructed.

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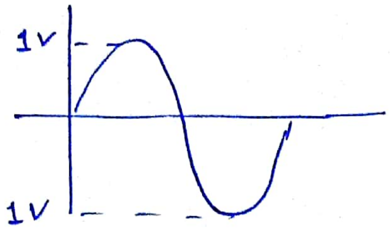
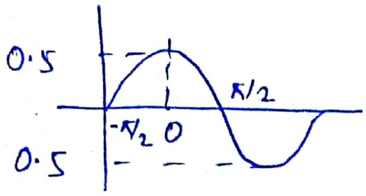
2 (B) : Given $\eta = 8$



$$P_s = \frac{A^2}{2} = \frac{1}{2} \text{ watt.}$$



$0.5 \cos \omega_m t$

$$P_s' = \frac{A^2}{2} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Here power is reduced by $\frac{1}{4}$ th ~~i.e~~

i.e power is reduced by 6 dB $\{10 \log 1/4 \simeq -6\,dB\}$

Therefore $(SN_qR)'_{dB}$ for 2 volts (peak to peak)

$$(SN_qR)'_{dB} = 6 \times 8 + 1.8 = 49.8\ dB.$$

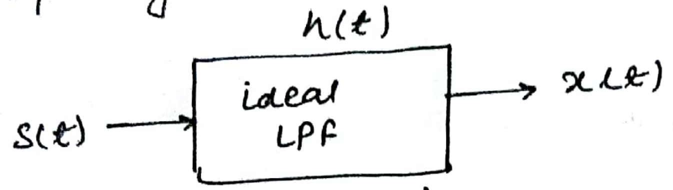
Hence, the $(SN_qR)_{dB}$ for input signal $0.5 \cos \omega_m t$ is

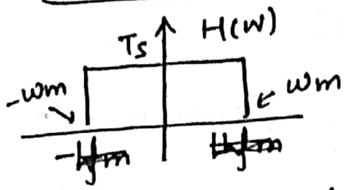
$$(SN_qR)_{dB} = (SN_qR)'_{dB} - 6\,dB$$
$$= (49.8 - 6)\ dB$$

$$\boxed{(SN_qR)_{dB} = 43.8\ dB}$$

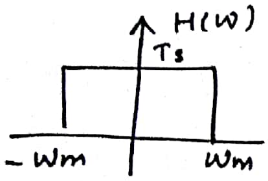
3. (A) Recovery of baseband signal from sampled signal

A signal ~~(band limited)~~ $x(t)$ band limited to f_m Hz can be reconstructed from its samples by passing the sampled signal through an ideal low pass filter having ~~a~~ ~~micro~~ cut-off frequency f_m Hz.



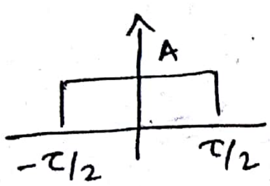


$$x(t) = \cancel{s(t)}\ s(t) * h(t) \quad \text{—— (A)}$$



$$\xleftrightarrow{F^{-1}} \quad h(t).$$

As we know



$$\Rightarrow A\,\text{rect}(t/\tau) \xrightarrow{F.T} A\tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

Using duality theorem

$$A\tau\, Sa\left(\frac{t\tau}{2}\right) \longleftrightarrow 2\pi A\, \underbrace{\text{rect}(-\omega/\tau)}_{\text{even function}}$$

$$\longrightarrow 2\pi A\, \text{rect}(\omega/\tau)$$

Now $\tau = 2\omega_m = \omega_s$

Also $2\pi A = T_s$

$$2\pi A = \frac{2\pi}{\omega_s} \Rightarrow A = 1/\omega_s$$

$$h(t) = A\tau\, Sa\left(\frac{t\tau}{2}\right) = \frac{T_s}{2\pi} \times \omega_s\, Sa\left(\frac{t\omega_s}{2}\right)$$

$$= \frac{\cancel{T_s}}{\cancel{2\pi}} \times \frac{\cancel{2\pi}}{\cancel{T_s}}\, Sa\left(\frac{\cancel{2}\omega_m t}{\cancel{2}}\right)$$

$$h(t) = Sa(\omega_m t) \quad \text{— (B)}$$

sampled signal $s(t)$ may be ~~te~~ written as

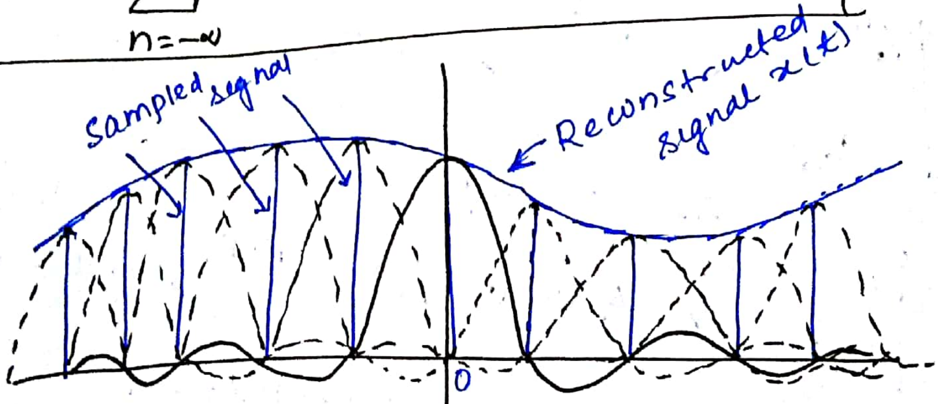
$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s) \quad \text{— (C)}$$

from (A), (B) & (C)

$$x(t) = s(t) * h(t)$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t-nT_s) * Sa(\omega_m t)$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, Sa\left(\omega_m (t - nT_s)\right)}$$



3 (B) :- $x(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$

$$= \frac{1}{4\pi} \{\cos(4000\pi + 1000\pi)t + \cos(4000\pi - 1000\pi)t\}$$

$$= \frac{1}{4\pi} \{\cos(5000\pi t) + \cos(3000\pi t)\}$$

Here $\omega_1 = 5000\pi$ $\quad \omega_2 = 3000\pi$

$f_1 = 2500$ $\quad f_2 = 1500$

maximum frequency present in the signal $f_1 = 2500\,Hz$

$\therefore$ Nyquist rate, $f_s = 2f_m = 2500 \times 2 = 5000\,Hz = 5\,kHz$

Nyquist interval, $T_s = 1/f_s = \frac{1}{5000} = 0.0002\,sec = 0.2\,msec.$

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4 (A) :- Entropy:- It is average information content of message or average information content per source symbol. It is denoted by H(x)

Mathematical expression of entropy
—o—

$$H(x) = \sum_{K=1}^{m} P_K \log_2\left(\frac{1}{P_K}\right)$$

where P_K is probability of occurance of K^{th} message

→ The unit of entropy is bits/symbol.

4 (B) Given:- M message, i.e $m_1, m_2, m_3 --- m_M$ are being transmitted having probability $P_1, P_2, P_3 --- P_K$.

⇒ Since the messages are equally likely, therefore probability of occurrance of each message should be $\frac{1}{M}$.

i.e $P_K = 1/M$.

Therefore, entropy for the given transmitting source 'X' is.

$$H(X) = \sum_{K=1}^{m} P_K \log_2\left(\frac{1}{P_K}\right) = \sum_{K=1}^{m} \frac{1}{M} \log_2\left(\frac{1}{1/M}\right)$$

$$= \sum_{K=1}^{m} \frac{1}{M} \log_2(M)$$

$$= \frac{1}{M}\log_2 M + \frac{1}{M}\log_2 M \text{ ---- upto M-times.}$$

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$$= \frac{M}{M} \log_2 M = \log_2 M.$$

$$\Rightarrow \boxed{H(x) = \log_2 M}$$

4(c) To be proved

$$0 \leq H(x) \leq \log_2 M.$$

As we know $H(x) = \sum_{K=1}^{M} P_K \log_2(1/P_K)$

Case 1 when $P_K = 1$; i.e event is sure.

then $H(x) = \sum_{K=1}^{m} P_K \log_2(1/P_K) = \sum_{K=1}^{m} 1 \times \log(1/1) = 0$

when $P_K = 0$;

then $H(x) = \sum_{K=1}^{m} 0 \times \log(1/0) = 0 \times K = 0$

$\therefore$ $\boxed{H(x) \geq 0}$ ← Lower bound

Proof of upper bound

Let us consider two probability distribution $[P(x_i) = P_i]$ and $[Q(x_i) = Q_i]$ on the alphabet $\{x_i\}$, where $i = 1, 2, 3 \ldots m$.

$$\therefore \sum_{i=1}^{m} P_i = 1 \quad \& \quad \sum_{i=1}^{m} Q_i = 1.$$

Now using inequality

$\ln(\alpha) \leq \alpha - 1$ where $\alpha > 0$

equality holds if $\alpha = 1$

$$\Rightarrow \sum_{i=1}^{m} P_i \ln\left(\frac{Q_i}{P_i}\right) \leq \sum_{i=1}^{m} P_i\left(\frac{Q_i}{P_i} - 1\right)$$

$$\leq \sum_{i=1}^{m} (Q_i - P_i) \leq \underbrace{\sum_{i=1}^{m} Q_i}_{1} - \underbrace{\sum_{i=1}^{m} P_i}_{1}$$

$$\Rightarrow \sum_{i=1}^{m} P_i \ln\left(\frac{Q_i}{P_i}\right) \leq 1 - 1 \leq 0$$

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$$\Rightarrow \sum_{i=1}^{m} p_i \ln\left(\frac{q_i}{p_i}\right) \leq 0 \Rightarrow \sum_{i=1}^{m} p_i \log_2\left(\frac{q_i}{p_i}\right) \leq 0$$

equality holds only $q_i = p_i$ for all 'i'.

Let $q_i = \frac{1}{m}$ $\quad i = 1, 2, 3 \ldots m$.

$$\sum_{i=1}^{m} p_i \log_2\left(\frac{1}{m p_i}\right) \leq 0$$

$$\Rightarrow \sum_{i=1}^{m} p_i \log_2\left(\frac{1}{p_i}\right) - \sum_{i=1}^{m} p_i \log_2 m \leq 0$$

$$\Rightarrow H(x) - \sum_{i=1}^{m} p_i \log_2 m \leq 0$$

$$\Rightarrow H(x) \leq \log_2 m \underbrace{\sum_{i=1}^{m} p_i}_{1}$$

$$\Rightarrow \boxed{H(x) \leq \log_2(M)}$$ ← upper bound.

equality hold when x are equiprobable.

<u>Hence</u> $\boxed{0 \leq H(x) \leq \log_2(M)}$

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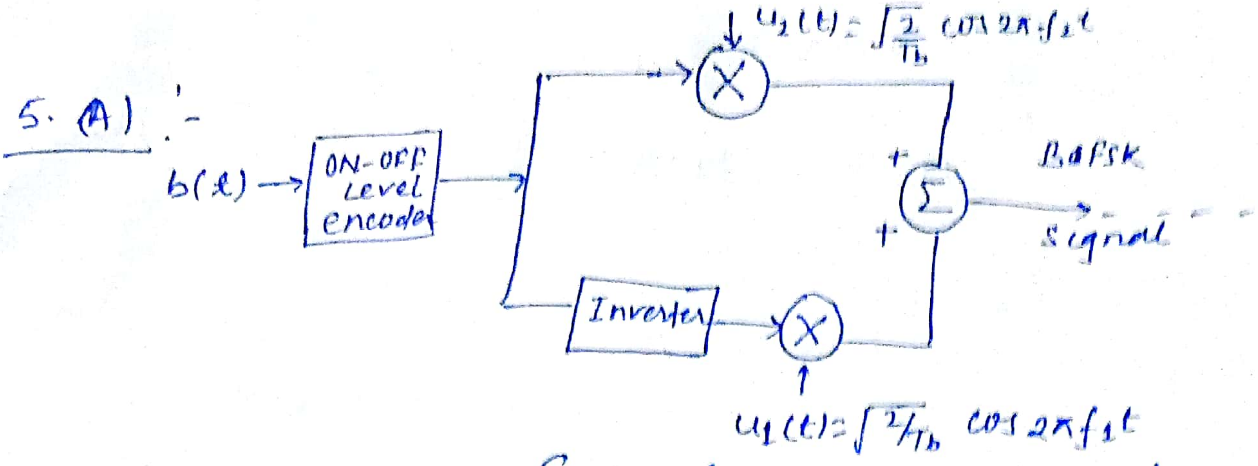
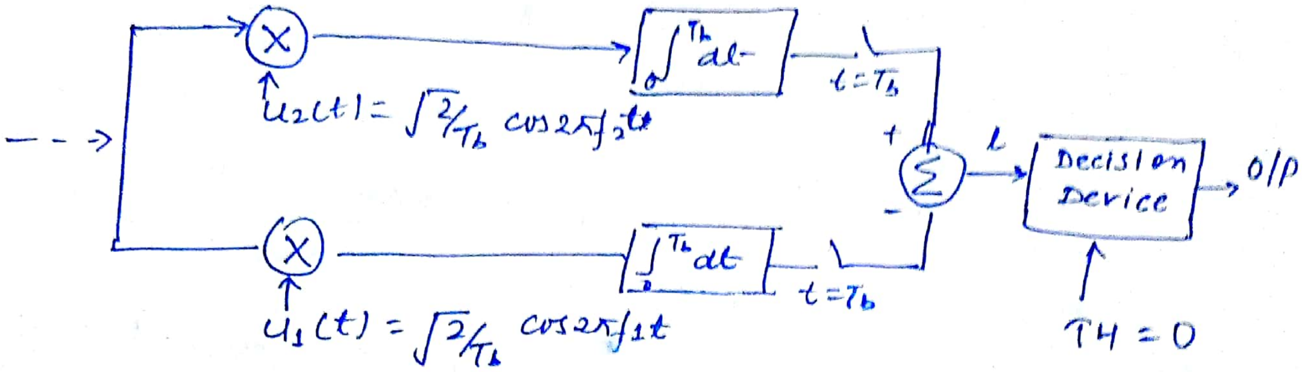
5. (A) :-



Generation (Transmitter)



Detection (Receiver)

choose '0' if $l < 0$
choose '1' if $l > 0$
choose random if $l = 0$

Transmission of '1'

$$\left\{ \underbrace{\int_0^{T_b} \sqrt{E_b}\sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \times \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t \, dt}_{=0} + \int_0^{T_b} \sqrt{E_b} \cdot \sqrt{2/T_b} \cos 2\pi f_2 t \cdot \cos 2\pi f_2 t \sqrt{2/T_b} \, dt \right\}$$

$$= \cancel{\sqrt{E_b}} \cancel{2} \times \cancel{1/2/T_b} = \sqrt{E_b} \frac{2}{T_b} \int_0^{T_b} \cos^2 2\pi f_2 t \, dt$$

$$= \sqrt{E_b} \frac{2}{T_b} \int_0^{T_b} \left(\frac{1 + \cos 4\pi f_2 t}{2} \right) dt = \sqrt{E_b} \times \frac{\cancel{2}}{\cancel{T_b}} \times \frac{\cancel{T_b}}{\cancel{2}}$$

$= \sqrt{E_b} \Rightarrow$ As $\sqrt{E_b} > 0$ Hence '1' was Transmitted

Transmission of '0'

$$\underbrace{\int_0^{T_b} \left(-\sqrt{E_b} \sqrt{2/T_b} \cos 2\pi f_1 t \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t \right) dt}_{=0} - \int_0^{T_b} \left(\sqrt{E_b} \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t \right) dt$$

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$$= -\sqrt{E_b}\sqrt{\frac{2}{T_b}}\int_0^{T_b}\left(\frac{1+\cos 4\pi f_1 t}{2}\right)dt = -\sqrt{E_b}\,\frac{2}{T_b}\times\frac{T_b}{2} = -\sqrt{E_b}$$

$\Rightarrow$ $l = -\sqrt{E_b} < 0$ Hence '0' was transmitted.

Working

1. The binary data sequence $b(t)$ is first applied to an optional ON-OFF level encoder to ensure that $b(t)$ is a unipolar NRZ waveform.

2. If $b(t) = 1$; then the higher frequency signal will be transmitted and if $b(t) = 0$; lower frequency signal will be transmitted. The inverter ensures that at a ~~any~~ time, only one signal will be transmitted.

3. The received BFSK signal is simultaneously applied to both the locally generated orthonormal functions but at a time only ~~& o/p~~ one output will be high depending upon whether '1' was transmitted or '0' was transmitted.

4. The output of product of 2 signals (Received signal & orthonormal signal) are fed to integrator and dump switch receiver which maximizes signal to noise ratio. At $t = T_b$; 2 samples x_1 & x_2. the two ~~signals~~ samples are substracted to generate resultant sample 'l' and is fed to the decision device.

5. The decision device decides between '0' & '1' depending upon its threshold value.

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5 (B) :- Given $R_b = 2.5 \times 10^6$ bits/sec.

PSD $= 10^{-20}$ watt/Hz

$\frac{\eta}{2} = 10^{-20} \Rightarrow \eta = 2 \times 10^{-20}$ (for 2-sided)

Amplitude of received sinusoidal signal

$= 1\ \mu$volt.

$$E_b = \frac{A^2}{2} \times T_b = \frac{A^2}{2} \times \frac{1}{R_b} \quad \left\{ T_b = \text{Pulse width} = \frac{1}{R_b} \right\}$$

$$= \frac{10^{-12}}{2} \times \frac{1}{2.5 \times 10^6} = 20 \times 10^{-20} \text{ watt-sec.}$$

Probability of error for FSK system.

$$P_{e(FSK)} = \frac{1}{2} \operatorname{erfc}\left\{ \sqrt{E_b/2\eta} \right\}$$

By using approximation :-

$$P_{e(FSK)} = \frac{1}{2} \frac{e^{-\sqrt{E_b/2\eta}}}{\sqrt{\pi}\sqrt{E_b/2\eta}}$$

$$\left\{ \frac{E_b}{2\eta} = \frac{20 \times 10^{-20}}{2 \times 2 \times 10^{-20}} = 5 \right\}$$

$$= \frac{1}{2} \frac{e^{-5}}{\sqrt{\pi \times 5}} = 8.5 \times 10^{-4}$$

$$\Rightarrow \boxed{P_{e(FSK)} = 8.5 \times 10^{-4}}$$

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6.(A) :- Shannon-Hartley law :- It states that the maximum amount of error free digital data that can be transmitted over a communication channel with a specified bandwidth in the presence of noise.

It can be expressed by the equation

$$\boxed{C = B \log_2 (1 + S/N)} \text{ bits/sec.}$$

where C - capacity of AWGN channel.

B - Bandwidth of channel.

S/N - signal to noise ratio.

Part II :- The channel capacity of an AWGN channel with infinite bandwidth is

$$C_\infty = 1.44 \, S/\eta \text{ bits/sec.}$$

As we know $N = \eta B$ —— (1)

N = noise power.

η : PSD of one sided

B = channel bandwidth.

Now, According to shannon hartley law :-

$$C = B \log_2 (1 + S/N) \text{ b/s}$$

$$= B \log_2 (1 + S/\eta B) \quad \text{from (1).}$$

Let $\frac{S}{\eta B} = x$, the above eqn can be written as.

$$C = \frac{S}{\eta x} \log_2(1+x) = \frac{1}{\ln 2} \frac{S}{\eta} \frac{\ln(1+x)}{x} \quad \text{—— (A)}$$

$$C_\infty = \lim_{B \to \infty} B \log_2 \left(1 + \frac{S}{\eta B}\right)$$

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If $B \to \infty$ $\quad x = \frac{S}{\eta B} \to 0$.

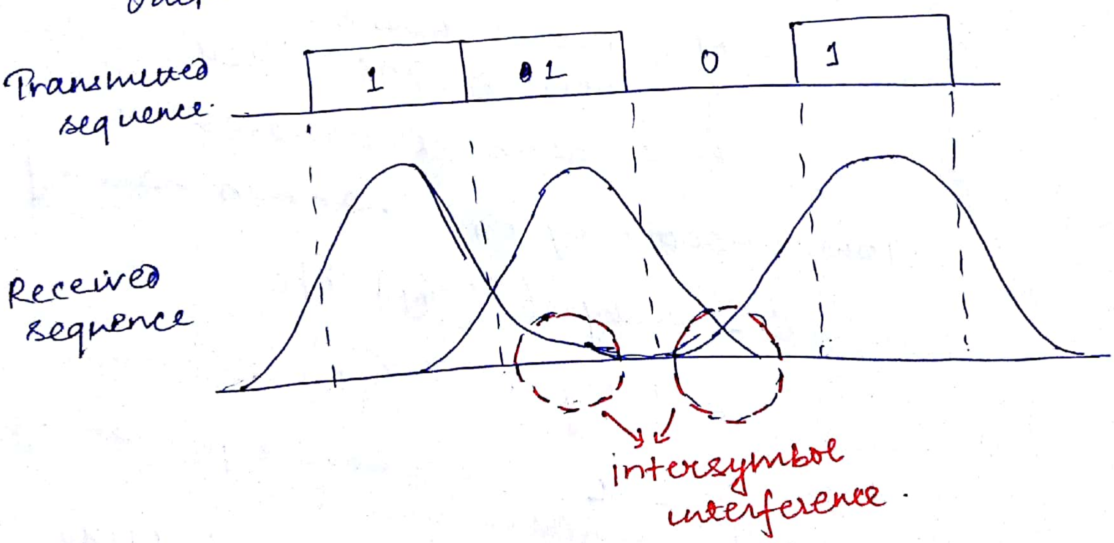
from eqn (A)

$$C_\infty = \frac{1}{\ln 2} \frac{S}{\eta} \lim_{x \to 0} \left(\frac{\ln(1+x)}{x} \right)$$

$$= \frac{1}{\ln 2} \frac{S}{\eta} \times 1 \qquad \left\{ \lim_{x \to 0} \left(\frac{\ln(1+x)}{x} \right) = 1 \right\}$$

$$\boxed{C_\infty = \frac{1}{\ln 2} \frac{S}{\eta} = 1.44\, S/\eta \text{ b/sec.}}$$

6.(B) Intersymbol interference (ISI) is a form of distortion of a signal in which one symbol interfere with adjacent symbol. causing noise or a less reliable signal. The main cause of intersymbol interference are multipath propagation or non linear frequency in channel. This has the effect of a blur or mixture of symbol, which can reduce signal clarity. If intersymbol interference occurs within a system, the receiver output becomes erroneous at the decision device.



This is an unfavourable result that should be reduced to the most minimum amount possible.

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ISI can be minimized/ eleminated by following ways.

1. Design a system such that the impulse response is short enough that very little energy from one symbol smears into next symbol.
2. Separate symbol in time with guard periods.
3. Apply an equilizer at the receiver, that broadly attempts to undo the effect of the channel by applying an inverse filter.
4. Apply a sequence detector at the receiver,

— x —o——o —x —

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