ANSWERS OF 'DSP' MID-SEM PAPER (NOV-2018)

1(i) Ans: (a)

$$g(n) = \left(\frac{1}{2}\right)^n u(n) = \left\{1, \frac{1}{2}, \frac{1}{2^2}, \ldots\right\}$$

$$x(n) = \{x_0, x_1, x_2, \ldots\}$$

$$y(n) = x(n) * g(n)$$

$$= \sum_{k=0}^{n} x(k) g(n-k)$$

given: $y(0) = 1$ & $y(1) = \frac{1}{2}$.

Now; $y(0) = x(0) \cdot g(0)$

$\Rightarrow 1 = x_0 \cdot 1 \Rightarrow \boxed{x_0 = 1}$

$y(1) = x(0) g(1) + x(1) g(0)$

$\Rightarrow \frac{1}{2} = 1 \times \frac{1}{2} + x_1 \cdot 1$

$\Rightarrow \boxed{x_1 = 0}$ Ans.

(ii) Ans: (b)

given: $y(n) = x(n-1)$

$Y(z) = z^{-1} X(z)$

$\Rightarrow \frac{Y(z)}{X(z)} = H(z) = z^{-1}$

Now, $H(z) = H_1(z) \cdot H_2(z)$

$\Rightarrow z^{-1} = \left[\frac{1 - 0.4z^{-1}}{1 - 0.6z^{-1}}\right] \cdot H_2(z)$

$\Rightarrow \boxed{H_2(z) = \frac{z^{-1}(1 - 0.6z^{-1})}{1 - 0.4z^{-1}}}$ Ans.

(iii) ~~(3)~~ Ans: (b)

For an N-point DFT of a real-valued sequence,

$$X_{N-K} = X_K^* \quad (N = 8 \text{ given})$$

$$\Rightarrow \left.\begin{array}{l} X_7 = X_1^* = 1+j3 \\ X_6 = X_2^* = 0 \end{array}\right\} \text{Ans.}$$

(iv) ~~(4)~~ Ans: (C)

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$= \frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - \frac{1}{2}z^{-1}} = H_1(z) + H_2(z)$$

Case(1): System is stable & causal.

$R_1: |z| > \frac{1}{4}$ & $R_2: |z| > \frac{1}{2}$

$\Rightarrow$ ROC: $R_1 \cap R_2 = |z| > \frac{1}{2}$

Hence S_1 is true.

Case(2): given ROC : $|z| < \frac{1}{4}$

This is possible only when $R_1: |z| < \frac{1}{4}$ and $R_2: |z| < \frac{1}{2}$

ROC doesn't include unity circle

$\Rightarrow$ S_2 is false.

Case(3): overall ROC: $\frac{1}{4} < |z| < \frac{1}{2}$

This means $R_1: |z| > \frac{1}{4}$ & $R_2: |z| < \frac{1}{2}$

Since ROC doesn't include unity circle, it is Unstable.

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Also, since R_2 lies interior to the circle, the system is non-causal

$\Rightarrow$ S_3 is TRUE.

(V) Ans: (a).

$h(n) = \{\underset{\uparrow}{0}, 0, 1, -1\}$ $\Rightarrow$ system is FIR (S_2 is true)

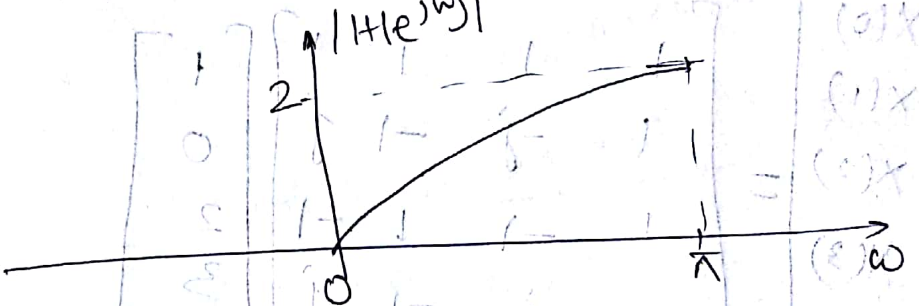
$\Rightarrow H(z) = z^{-2} - z^{-3}$

$\Rightarrow H(e^{jw}) = e^{-j2w} - e^{-j3w}$

$= e^{-j2.5w}\left(\frac{e^{+j0.5w} - e^{-j0.5w}}{j2}\right) j2$

$= j2 e^{-j2.5w} \sin\left(\frac{w}{2}\right)$

$\Rightarrow |H(e^{jw})| = 2\left|\sin\frac{w}{2}\right|$



From freq. response; the system is High Pass filter. $\Rightarrow$ S_1 is false.

2(i) $x(n) = \{1, 0, 2, 3\}$

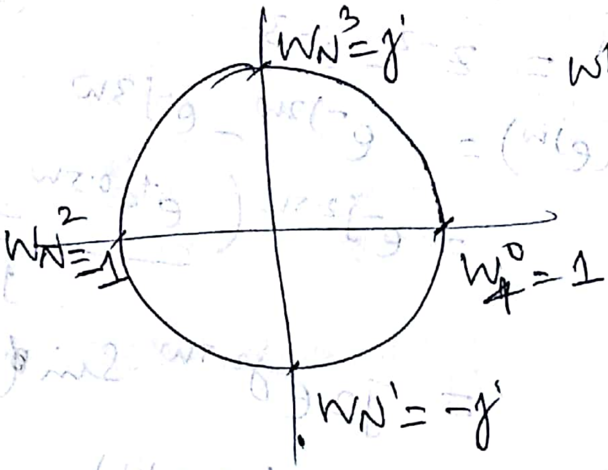
~~$X(k) = W_4\, x(n)$~~

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

PTO

Using twiddle matrix for N=4.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$



where $N = 4$ & $W_N = e^{-j\frac{2\pi}{4}}$

$$\Rightarrow \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+2+3 \\ 1-0-2+3j \\ 1-0+2-3 \\ 1+0-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ -1+3j \\ 0 \\ -1-3j \end{bmatrix}$$

$\Rightarrow X(k) = \{6, -1+3j, 0, -1-3j\}$ Ans.

2(ii) $R = 200k$; $C = 10\,\mu F \Rightarrow RC = 200 \times 10^3 \times 10 \times 10^{-6}$

$= 2$

$V_i(t) = 5V$ DC .

$\Rightarrow X(s) = \frac{R}{R + \frac{1}{sC}} \cdot V_i(s)$

$= \frac{sRC}{sRC+1} \cdot \frac{5}{s}$

$\Rightarrow X(s) = \frac{5}{s + 1/RC} = \frac{5}{s+0.5}$

$\Rightarrow x(t) = 5e^{-0.5t}$.

Sampling Rate, $f_s = 10\,Hz$.

$\Rightarrow T_s = \frac{1}{f_s} = 0.1$.

$\Rightarrow x(n) = x(t)\big|_{t = nT_s}$

$= 5e^{-0.5 \times 0.1 n}$

or, $\boxed{x(n) = 5e^{-0.05n}}$ Ans.

$X(z) = Z\{x(n)\}$

or, $\boxed{X(z) = \frac{5}{1 - e^{-0.05} \cdot z^{-1}}}$ Ans.

5(i) $x(n) \xleftrightarrow{Z} X(z)$

$\Rightarrow$ $$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

differentiating both sides w.r.t. 'z'.

$\Rightarrow$ $$\frac{dX(z)}{dz} = \sum_{n=0}^{\infty} x(n) \{-n z^{-n-1}\}$$

$$= -z^{-1} \sum_{n=0}^{\infty} n x(n) z^{-n}$$

$\Rightarrow$ $$\sum_{n=0}^{\infty} n x(n) z^{-n} = -z \frac{dX(z)}{dz}$$

or, $$\boxed{Z\{n x(n)\} = -z \frac{dX(z)}{dz}}$$ Ans.

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5 (ii) $x(n) = \alpha^n u(n)$; $|\alpha| < 1$

$= 1 + \alpha + \alpha^2 + \cdots$

$\Rightarrow$ $$X(e^{jw}) = \sum_{n=0}^{\infty} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{\infty} \alpha^n e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n$$

$$= 1 + (\alpha e^{-jw}) + (\alpha e^{-jw})^2 + \cdots$$

$\Rightarrow$ $$\boxed{X(e^{jw}) = \frac{1}{1-\alpha e^{-jw}}}$$

~~$1 \neq \alpha e^{jw}$~~

Now, $$X(e^{jw}) = \frac{1}{1-\alpha \cos w + j\alpha \sin w}$$

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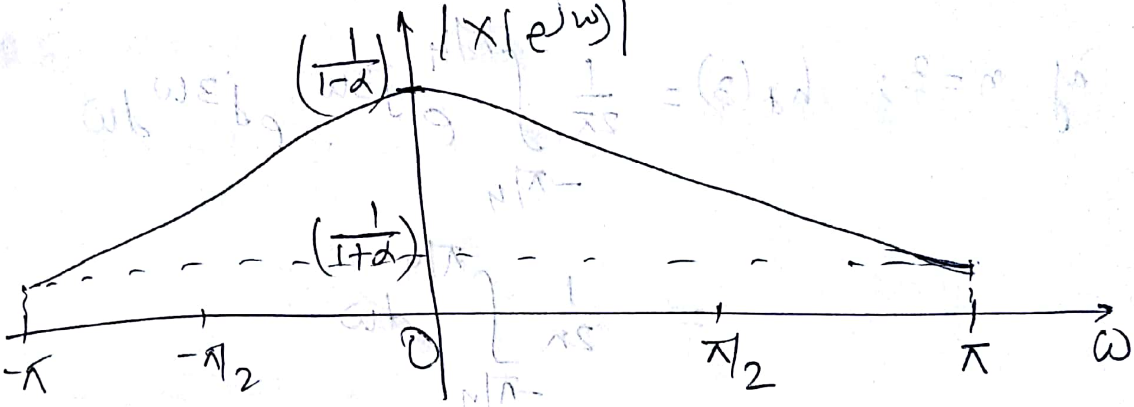
$$\Rightarrow \cancel{|X(e^{j\omega})| = \frac{1}{\sqrt{(1-\alpha\cos\omega)^2 + (\alpha\sin\omega)^2}}}$$

$$\Rightarrow |X(e^{j\omega})| = \frac{1}{\sqrt{(1-\alpha\cos\omega)^2 + (\alpha\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1+\alpha^2 - 2\alpha\cos\omega}}.$$



(4.) $H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; -\pi/4 \le \omega \le \pi/4 \\ 0 & ; \text{e.w.} \end{cases}$

desired freq. response: $H_d(e^{j\omega})$

Length of filter, $M = 7$.

cut off freq, $\omega_c = \pi/4$

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j3\omega}\cdot e^{j\omega n}\, d\omega$$

$$\cancel{= \frac{1}{2\pi}\Big| e^{j(n-3)}}$$

P.T.O

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\left|\frac{e^{j\omega(n-3)}}{j(n-3)}\right|_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi}\left(\frac{e^{j(n-3)\pi/4} - e^{-j(n-3)\pi/4}}{j(n-3)}\right)$$

$$= \frac{\sin(n-3)\pi/4}{\pi(n-3)} \quad ; n \neq 3$$

if $n=3$; $h_d(3) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j3\omega} \cdot e^{j3\omega}\, d\omega$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} d\omega$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{4} - (-\pi/4)\right]$$

$$= \frac{1}{2\pi}\left(\frac{\pi}{2}\right) = \frac{1}{4}.$$

$$\Rightarrow h_d(n) = \begin{cases} \frac{1}{4} & ; n = 3 \\ \frac{\sin(n-3)\pi/4}{\pi(n-3)} & ; n \neq 3 \end{cases}$$

$$\Rightarrow h_d(0) = -\frac{\sin(3\pi/4)}{-3\pi} = \frac{1}{3\pi} \times \frac{1}{\sqrt{2}} = 0.075$$

$$h_d(1) = \frac{-\sin 2\pi/4}{-2\pi} = \frac{1}{2\pi} \times 1 = 0.159$$

$$h_d(2) = \frac{-\sin \pi/4}{-\pi} = \frac{1}{\pi} \times \frac{1}{\sqrt{2}} = 0.225$$

$$hd(3) = \frac{1}{4} = 0.25$$

$$hd(4) = \frac{\sin(\pi/4)}{\pi} = \frac{1}{\pi} \times \frac{1}{\sqrt{2}} = 0.225$$

$$hd(5) = \frac{\sin(2\pi/4)}{2\pi} = \frac{1}{2\pi} \times 1 = 0.159$$

$$hd(6) = \frac{\sin(3\pi/4)}{3\pi} = \frac{1}{3\pi} \times \frac{1}{\sqrt{2}} = 0.075$$

- For Hamming Window;

$$W_H(n) = \begin{cases} 0.54 - 0.46 \cos \frac{2\pi n}{M-1} & \text{for } n = 0, 1, 3 \ldots M-1 \\ 0 & i.e.w. \end{cases}$$

given $M = 7$

$\Rightarrow W_H(0) = 0.54 - 0.46 = 0.12$

$W_H(1) = 0.54 - 0.46 \cos \frac{2\pi}{6} = 0.54 - 0.46 \times \frac{1}{2} = 0.31$

$W_H(2) = 0.54 - 0.46 \cos \frac{2\pi \times 2}{6} = 0.54 + 0.46 \times \frac{1}{2} = 0.77$

$W_H(3) = 0.54 - 0.46 \cos \frac{2\pi \times 3}{6} = 0.54 + 0.46 = 1$

$W_H(4) = 0.54 - 0.46 \cos \frac{2\pi \times 4}{6} = 0.54 + 0.46 \times \frac{1}{2} = 0.77$

$W_H(5) = 0.54 - 0.46 \cos \frac{2\pi \times 5}{6} = 0.54 - 0.46 \times \frac{1}{2} = 0.31$

$W_H(6) = 0.54 - 0.46 \cos \frac{2\pi \times 6}{6} = 0.54 - 0.46 = 0.12$

Now; $h(n) = hd(n) \cdot w(n)$ for $n = 0, 1, 2, \ldots, M-1$

PTO

$\Rightarrow h(0) = h_d(0) \cdot w(0) = 0.075 \times 0.12 = 0.009$

$h(1) = h_d(1) \cdot w(1) = 0.159 \times 0.31 = 0.049$

$h(2) = h_d(2) \cdot w(2) = 0.225 \times 0.77 = 0.173$

$h(3) = h_d(3) \cdot w(3) = 0.25 \times 1 = 0.25$

$h(4) = h_d(4) \cdot w(4) = 0.225 \times 0.77 = 0.173$

$h(5) = h_d(5) \cdot w(5) = 0.159 \times 0.31 = 0.049$

$h(6) = h_d(6) \cdot w(6) = 0.075 \times 0.12 = 0.009$.

$\Rightarrow \boxed{h(n) = \{0.009, 0.049, 0.173, 0.25, 0.173, 0.049, 0.009\}}$

Ans.

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(3) given: $0.9 \leq H(e^{j\omega}) \leq 1$; $0 \leq \omega \leq \pi/2$

$H(e^{j\omega}) \leq 0.2$; $\frac{3\pi}{4} \leq \omega \leq \pi$

$T_s = 1$ sec.

Hence; $\omega_p = \pi/2$

& $\omega_s = 3\pi/4$.

Using Bilinear transformation;

$$\Omega = \frac{2}{T_s} \tan\frac{\omega}{2}$$

$$\Rightarrow \Omega_p = \frac{2}{1} \tan\frac{\pi/2}{2} = 2$$

$$\& \ \Omega_s = \frac{2}{1} \tan\frac{3\pi/4}{2} = \cancel{1.1775}\ 4.828$$

Therefore specifications of analog filter are as follows:

$A_p = 0.9$; $\Omega_p = 2$

$A_s = 0.2$; $\Omega_s = \cancel{1.1775}\ 4.828$

Calculation of order 'N':

$$N \geq \frac{1}{2} \times \frac{\log\left[\left(\frac{1}{A_s^2}-1\right)/\left(\frac{1}{A_p^2}-1\right)\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\geq \frac{1}{2} \times \frac{\log\left[\left(\frac{1}{0.2^2}-1\right)/\left(\frac{1}{0.9^2}-1\right)\right]}{\log\left(\frac{4.828}{2}\right)}$$

$$\geq \frac{1}{2} \times \frac{\log\left[\frac{24}{0.2345}\right]}{\log\left[2.414\right]}$$

$$\geq \frac{1}{2} \times \frac{2}{0.3827} = 2.613$$

$$\Rightarrow \boxed{N = 3}$$

Determination of cut-off freq, Ω_c:

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2}-1\right)^{1/2N}} = \frac{2}{\left(\frac{1}{0.9^2}-1\right)^{\frac{1}{2\times 3}}}$$

$$= \frac{2}{(0.2345)^{1/6}} = \frac{2}{0.7853} = 2.5468$$

PTO

Determination of poles:

$$p_i = \pm \Omega_c \, e^{j(N+2i+1)\pi/2N} \quad ; \; i = 0, 1, \ldots N-1$$

Since $N = 3$

$\Rightarrow \quad p_0 = \pm 2.5468 \, e^{j(3+0+1)\pi/6}$

$= \pm 2.5468 \left(\cos \frac{2\pi}{3} + j \sin \frac{2\pi}{3} \right)$

$= \pm 2.5468 \left(-0.5 + j \, 0.707 \right)$

$= \pm \left(1.2734 + j1.8 \right)$

$p_1 = \pm 2.5468 \, e^{j(3+2+1)\pi/6}$

$= \pm 2.5468 \left(\cos \pi + j \sin \pi \right)$

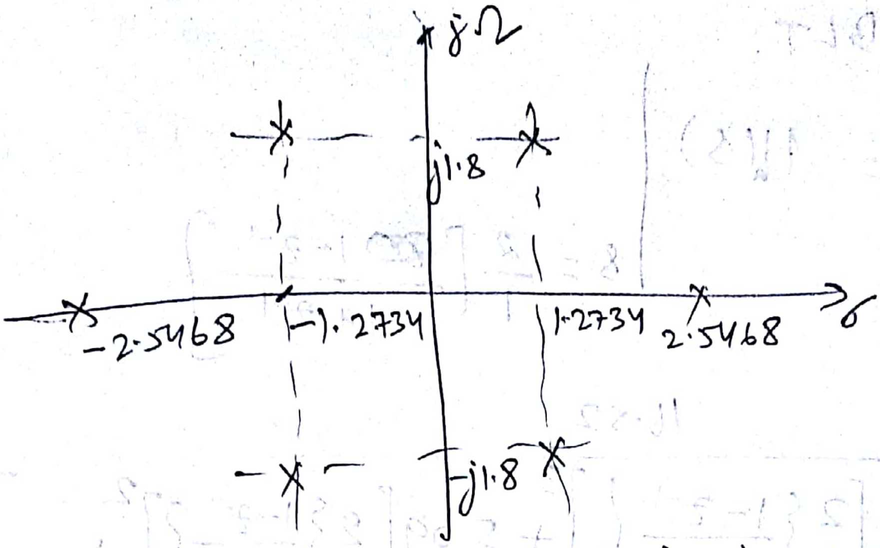
$= \pm 2.5468$

$p_2 = \pm 2.5468 \, e^{j(3+6+1)\frac{\pi}{6}}$

$= \pm 2.5468 \left(\cos \frac{5\pi}{3} + j \sin \frac{5\pi}{3} \right)$

$= \pm 2.5468 \left(0.5 - j \, 0.707 \right)$

$= \pm \left(1.2734 - j \, 1.8 \right)$



Considering only the poles lying in LH of s-plane;

poles are at: -2.5468 ; $-1.2734 \pm j1.8$.

Determination of Ha(s):

$$H_a(s) = \frac{\Omega_c^N}{(s-s_0)(s-s_1)(s-s_2)}$$

$$= \frac{(2.5468)^3}{(s+2.5468)\{(s+1.2734)^2 + (1.8)^2\}}$$

$$= \frac{16.52}{(s+2.5468)\{s^2 + 2.5468s + 1.622 + 3.24\}}$$

$$= \frac{16.52}{(s+2.5468)(s^2 + 2.5468s + 4.862)}$$

$$= \frac{16.52}{s^3 + 5.09s^2 + 9.96s + 12.38}$$

$\boxed{\text{PTO}}$

Using BLT:

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{16.52}{\left[2\left\{\frac{1-z^{-1}}{1+z^{-1}}\right\}\right]^3 + 5.09\left[2\left\{\frac{1-z^{-1}}{1+z^{-1}}\right\}\right]^2 + 9.98\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right\} + 12.32}$$

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