MUZAFFARPUR INSTITUTE OF TECHNOLOGY, MUZAFFARPUR

Mid-Term Examination, 2018

Session 2018- 19 (Odd Semester)

Subject: Strength of Materials **Paper code: 021306**

Course & Semester: B.Tech (3^{rd}) **Branch: ME**

Solution

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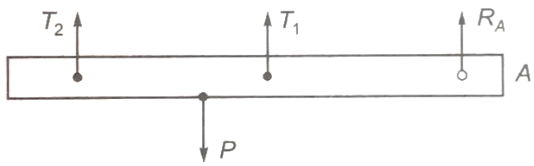
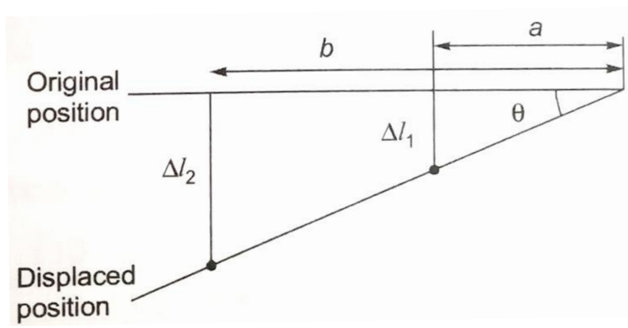
1. Drawing free body diagram.



R_A is the reaction force at A. Let the rigid bar attains equilibrium at an angle θ from the horizontal position.



Δl_2 = change in length of steel wire carrying tension T_2

Δl_1 = change in length of steel wire carrying tension T_1

$\Delta l = \frac{PL}{AE}$

$\therefore \Delta l_2 = \frac{T_2 L}{AE}, \Delta l_1 = \frac{T_1 L}{AE}$

$\therefore \tan\theta = \frac{\Delta l_2}{b} = \frac{\Delta l_1}{a}$ or $\Delta l_2 = \frac{b}{a}\Delta l_1$

or $\frac{T_2 L}{AE} = \frac{b}{a} \text{ x } \frac{T_1 L}{AE}$ or $T_2 = \frac{b}{a} T_1$

At equilibrium summation of all moments about point A will be zero.

$\therefore \quad \Sigma M_A = T_2 b + T_1 a - Pl = 0$

$\therefore \quad T_2 b + T_1 a = Pl$

$(\frac{b}{a} T_1)\, b + T_1 a = Pl$

$\frac{T_1 b^2}{a} + T_1 a = Pl$

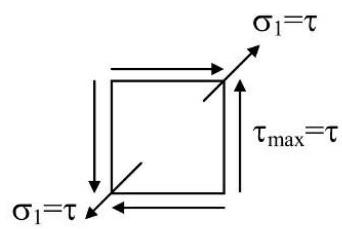
$T_1 b^2 + T_1 a^2 = Pal$

$$\mathbf{T_1} = \frac{\boldsymbol{Pal}}{\boldsymbol{a^2 + b^2}}$$

Therefore, $T_2 = \frac{b}{a} T_1$

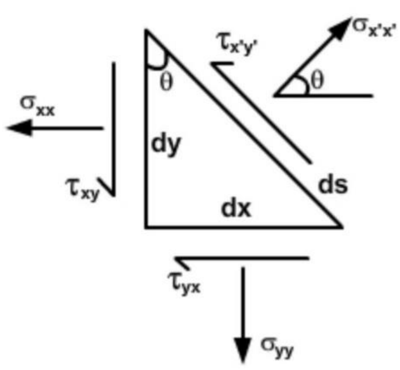
$$\mathbf{T_2} = \frac{\boldsymbol{Pbl}}{\boldsymbol{a^2 + b^2}}$$

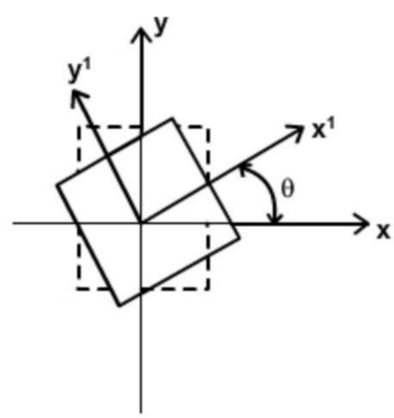
2. $\frac{\tau_{max}}{\sigma_1} = 1$



3. Though the state of stress at a point in a stressed body remains the same, the normal and shear stress components vary as the orientation of plane through that point changes. Under complex loading, a structural member may experience larger stresses on inclined planes then on the cross section.

The knowledge of maximum normal and shear stresses and their plane's orientation assumes significance from failure point of view. Hence, it is important to know how to transform the stress components from one set of coordinate axes to another set of co-ordinates axes that will contain the stresses of interest.





Consider a prismatic element with sides dx, dy and ds with their face perpendicular to y, x and x' axes respectively. Thickness of the element is t.

$\sigma_{x'x'}$ and $\tau_{x'y'}$ are the normal and shear stresses acting on a plane inclined at an angle θ measured counter clockwise from x plane.

Under equilibrium, $\sum F_{x'} = 0$

$$\sigma_{x'x'}.t.ds - \sigma_{xx}.t.d_y.\cos\theta - \sigma_{yy}.t.d_x.\sin\theta - \tau_{xy}.t.d_y.\sin\theta - \tau_{yx}.t.d_x.\cos\theta = 0$$

Dividing above equation by t.ds and using dy/dx = cosθ and dx/ds = sinθ,

$$\sigma_{x'x'} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\,\tau_{yx}\sin\theta\cos\theta$$

Similarly, from $\sum F_{y'} = 0$ and simplifying,

$$\tau_{x'y'} = (\sigma_{yy} - \sigma_{xx})\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Using trigonometric relations and simplifying,

$$\sigma_{x'x'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \ldots\textbf{(1}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \quad \ldots\textbf{(2}$$

Replacing θ by $\theta + 90^0$, in $\sigma_{x'x'}$ expression of equation 1, we get the normal stress along y' direction.

$$\sigma_{y'y'} = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \ldots\textbf{(3}$$

Equations 1 and 2 are the transformation equations for plane stress using which the stress components on any plane passing through the point can be determined. Notice here that,

$$\boldsymbol{\sigma_{xx} + \sigma_{yy} = \sigma_{x'x'} + \sigma_{y'y'}}$$

Invariably, the sum of the normal stresses on any two mutually perpendicular planes at a point has the same value. This sum is a function of the stress at that point and not on the orientation of axes.

4. See the class notebook.

5. Generalized Hook’s law:

Generalised Hook's law:-

Stress component

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \gamma_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Strain Component:-

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_{zz} \end{bmatrix}$$

Assumptions:

- Applicable upto proportionality limit.
- No impact loading
- Temperature should be constant.
- Material should be homogenous (Plastics, alloy, paper) and isotropic (glass, metals)

Anisotropic or Aelotropic Material:- A material having a mechanical property which has a different value when measured in different directions.

e.g woods and composits.

Orthotropic Materials:- Orthotropic materials have mechanical properties that differ along three mutually orthogonal axes of rotational symmetry.

e.g. Wood is an example of an orthotropic material material properties in three perpendicular directions (axial, radial and circumferential) are different.

orthotropic materials are a subset of anisotropic materials, because their properties change when measured from different directions.

Cubic Materials :- If the mechanical properties are same in X-Y ~~and~~ Y-Z and Z-X plane. Then the material is known as cubic material.

e.g. most of the crystalline solids like pure metals, salt (NaCl).

The generalised Hook's law for a material is given as :-

$$\sigma_{xx} = C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + C_{14}\gamma_{xy} + C_{15}\gamma_{yz} + C_{16}\gamma_{zx} + C_{17}\gamma_{yx} + C_{18}\gamma_{zy} + C_{19}\gamma_{xz}$$

$$\sigma_{yy} = C_{21}\varepsilon_{xx} + C_{22}\varepsilon_{yy} + C_{23}\varepsilon_{zz} + C_{24}\gamma_{xy} + C_{25}\gamma_{yz} + C_{26}\gamma_{zx} + C_{27}\gamma_{yx} + C_{28}\gamma_{zy} + C_{29}\gamma_{xz}$$

$$\sigma_{zz} = C_{21}\varepsilon_{xx} + C_{32}\varepsilon_{yy} + C_{33}\varepsilon_{zz} + C_{34}\gamma_{xy} + C_{35}\gamma_{yz} + C_{36}\gamma_{zx} + C_{37}\gamma_{yx} + C_{38}\gamma_{zy} + C_{39}\gamma_{xz}$$

$$\tau_{xy} = C_{41}\varepsilon_{xx} + C_{42}\varepsilon_{yy} + C_{43}\varepsilon_{zz} + C_{44}\gamma_{xy} + C_{45}\gamma_{yz} + C_{46}\gamma_{zx} + C_{47}\gamma_{yx} + C_{48}\gamma_{zy} + C_{49}\gamma_{xz}$$

$$\tau_{yz} = C_{51}\varepsilon_{xx} + C_{52}\varepsilon_{yy} + C_{55}\varepsilon_{zz} + C_{54}\gamma_{xy} + C_{55}\gamma_{yz} + C_{56}\gamma_{zx} + C_{57}\gamma_{yx} + C_{58}\gamma_{zy} + C_{59}\gamma_{xz}$$

$$\tau_{zx} = C_{61}\varepsilon_{xx} + C_{62}\varepsilon_{yy} + C_{63}\varepsilon_{zz} + C_{64}\gamma_{xy} + C_{65}\gamma_{yz} + C_{66}\gamma_{zx} + C_{67}\gamma_{yx} + C_{68}\gamma_{zy} + C_{69}\gamma_{xz}$$

$$\tau_{yx} = C_{71}\varepsilon_{xx} + C_{72}\varepsilon_{yy} + C_{73}\varepsilon_{zz} + C_{74}\gamma_{xy} + C_{75}\gamma_{yz} + C_{76}\gamma_{zx} + C_{77}\gamma_{yx} + C_{78}\gamma_{zy} + C_{79}\gamma_{xz}$$

$$\tau_{zy} = C_{81}\varepsilon_{xx} + C_{82}\varepsilon_{yy} + C_{83}\varepsilon_{zz} + C_{84}\gamma_{xy} + C_{85}\gamma_{yz} + C_{86}\gamma_{zx} + C_{87}\gamma_{yx} + C_{88}\gamma_{zy} + C_{89}\gamma_{xz}$$

$$\tau_{xz} = C_{91}\varepsilon_{xx} + C_{92}\varepsilon_{yy} + C_{93}\varepsilon_{zz} + C_{94}\gamma_{xy} + C_{95}\gamma_{yz} + C_{96}\gamma_{zx} + C_{97}\gamma_{yx} + C_{98}\gamma_{zy} + C_{99}\gamma_{xz}$$

From above expression

σ_{ii} σ_{jj} σ_{zz} $\rightarrow$ First order tensor

σ_{ij} $\rightarrow$ second order tensor

The individual elements of this tensor are the stifness ~~matrix, as popularly~~ known. Coefficient for this linear stress - strain relationship.

The stifness tensor has 81 independent element.

Each has individual elastic constants.

Total elastic constant for anistropic material = 81

Now, This value can be reduced by different symmetry.

① stress symmetry:

We know that

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zx}$$

$$\tau_{zx} = \tau_{xz}$$

Thus, with this symmetry the number of independent elastic constants reduces to (6×9 => 54 from 81.

② strain symmetry:-

$$\gamma_{xy} = \gamma_{yx}$$

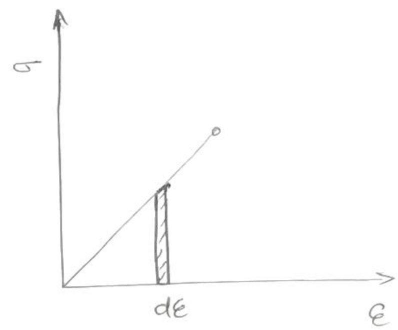
$$\gamma_{yz} = \gamma_{zx}$$

$$\gamma_{zx} = \gamma_{xz}$$

Thus, with this symmetry the number of independent elastic constants reduces to (6×6) 36 from 54.

(3) Now, Symmetry ~~strain~~ energy :-

Strain energy for this element



$$du = \sigma . d\varepsilon$$

If all the six component of stresses are acting

Then,

$$du = \sigma_{xx} d\varepsilon_{xx} + \sigma_{yy}\, d\varepsilon_{yy} + \sigma_{zz}\, d\varepsilon_{zz} + \tau_{xy} . d\gamma_{xy} + \tau_{yz} d\gamma_{yz} + \tau_{zx} . d\gamma_{zx}$$

Now for general case

$$\frac{\partial u}{\partial \varepsilon_{xx}} = \sigma_{xx} \qquad \frac{\partial u}{d\varepsilon_{yy}} = \sigma_{yy} \qquad \frac{\partial u}{\partial \varepsilon_{zz}} = \sigma_{zz}$$

$$\frac{\partial u}{\partial \gamma_{xy}} = \tau_{xy} \qquad \frac{\partial u}{\partial \gamma_{yz}} = \tau_{yz} \qquad \frac{\partial u}{\partial \gamma_{zx}} = \tau_{zx}$$

Therefore,

$$\sigma_{xx} = \frac{\partial u}{\partial \varepsilon_{xx}} = C_{11}\,\varepsilon_{xx} + C_{12}\,\varepsilon_{yy} + C_{13}\,\varepsilon_{zz} + C_{14}\,\gamma_{xy} + C_{15}\,\gamma_{yz} + C_{16}\,\gamma_{zx}$$

$$\sigma_{yy} = \frac{\partial u}{\partial \varepsilon_{yy}} = C_{21}\,\varepsilon_{xx} + C_{22}\,\varepsilon_{yy} + C_{23}\,\varepsilon_{zz} + C_{24}\,\gamma_{xy} + C_{25}\,\gamma_{yz} + C_{26}\,\gamma_{zx}$$

$$\sigma_{zz} = \frac{\partial u}{\partial \varepsilon_{zz}} = C_{31}\,\varepsilon_{xx} + C_{32}\,\varepsilon_{yy} + C_{33}\,\varepsilon_{zz} + C_{34}\,\gamma_{xy} + C_{35}\,\gamma_{yz} + C_{26}\,\gamma_{zx}$$

$$\tau_{xy} = \frac{\partial u}{\partial \gamma_{xy}} = C_{41}\,\varepsilon_{xx} + C_{42}\,\varepsilon_{yy} + C_{43}\,\varepsilon_{zz} + C_{44}\,\gamma_{xy} + C_{45}\,\gamma_{yz} + C_{46}\,\gamma_{zx}$$

$$\tau_{yz} = \frac{\partial u}{\partial \gamma_{yz}} = C_{51}\,\varepsilon_{xx} + C_{52}\,\varepsilon_{yy} + C_{53}\,\varepsilon_{zz} + C_{54}\,\gamma_{xy} + C_{55}\,\gamma_{yz} + C_{56}\,\gamma_{zx}$$

$$\tau_{zx} = \frac{\partial u}{\partial \gamma_{zx}} = C_{61}\,\varepsilon_{xx} + C_{62}\,\varepsilon_{yy} + C_{63}\,\varepsilon_{zz} + C_{64}\,\gamma_{xy} + C_{65}\,\gamma_{yz} + C_{66}\,\gamma_{zx}$$

differentiating above equations

$$\frac{\partial^2 u}{\partial \epsilon_{xx} \cdot \partial \epsilon_{yy}} = C_{12}$$

$$\frac{\partial^2 u}{\partial \epsilon_{yy} \cdot \partial \epsilon_{xx}} = C_{21}$$

Therefore $\boxed{C_{12} = C_{21}}$

Similarly

$C_{61} = C_{16}$, $C_{13} = C_{31}$, $C_{14} = C_{41}$, $C_{51} = C_{15}$

and so on.

These above relations are known as Complaince relation.

In matrix form.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

After considering strain energy symmetry.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

This symmetric matrix has 21 independent elastic Constants.

Therefore, we can say that Anisotropic material has 21 independent elastic constants.

④ Material symmetry:-

a. for orthotropic materials:

9 independent elastic constants.

b. for cubic material:

3 independent elastic constants.

c. Isotropic Materials:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{11} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

Two independent elastic constant.

$$\left.\begin{aligned} C_{11} &= 2G + \lambda \\ C_{12} &= \lambda \end{aligned}\right]$$

where $\lambda \rightarrow$ Lami's constant

$$\Rightarrow \sigma_{xx} = (2\lambda + G)\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{yy} = \lambda\,\varepsilon_{xx} + (2\lambda + G)\,\varepsilon_{yy} + \lambda\,\varepsilon_{zz}$$

$$\sigma_{zz} = \lambda\,\varepsilon_{xx} + \lambda\,\varepsilon_{yy} + (2\lambda + G)\,\varepsilon_{zz}$$

$$\tau_{xy} = G\,\varepsilon_{xy}, \quad \tau_{yz} = G\,\varepsilon_{yz}, \quad \tau_{zx} = G\,\varepsilon_{zx}$$

6.

Solution. Power to be transmitted = 75 kW

Speed, $N = 200$ r.p.m.

Allowable shear stress, $\tau = 70\ \text{MN/m}^2$

$$T_{max} = 1{\cdot}3\ T_{mean}$$

Using the relation: $P = \dfrac{2\pi NT}{60 \times 1000}$, we get

$$75 = \frac{2\pi \times 200 \times T}{60 \times 1000}$$

$$T = (T_{\text{mean}}) = \frac{75 \times 60 \times 1000}{2\pi \times 200} = 3581\ \text{Nm}$$

$$T_{max} = 1{\cdot}3 \times 3581 = 4655{\cdot}3\ \text{Nm}$$

Also,

$$T = \tau \times \frac{\pi}{16} \times D^3$$

$$4655{\cdot}3 = 70 \times 10^6 \times \frac{\pi}{16} \times D^3$$

$$D^3 = \frac{4655{\cdot}3 \times 16}{70 \times 10^6 \times \pi} = 338{\cdot}7 \times 10^{-6}$$

$\therefore$ $D = 0{\cdot}0697$ m or **69.7 mm** **(Ans.)**

7.https://nptel.ac.in/courses/Webcourse-contents/IIT-ROORKEE/strength%20of%20materials/lects%20&%20picts/image/lect16/lecture16.htm