Solution Mid-term Test

5th Sem, Network Theory, Electrical Engg.

1. (i)

To find Laplace transform of $f(t) = e^{-at}\sin\omega t$

Putting $\sin\omega t = \dfrac{e^{j\omega t} - e^{-j\omega t}}{2j}$

$$\Rightarrow f(t) = e^{-at}\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right]$$

$$= \frac{1}{2j}\left[e^{-(a-j\omega)t} - e^{-(a+j\omega)t}\right]$$

$$\mathcal{L}f(t) = \frac{1}{2j}\left[\frac{1}{s+(a-j\omega)} - \frac{1}{s+(a+j\omega)}\right]$$

$$\Rightarrow \boxed{\mathcal{L}f(t) = \frac{\omega}{(s+a)^2+\omega^2}}$$

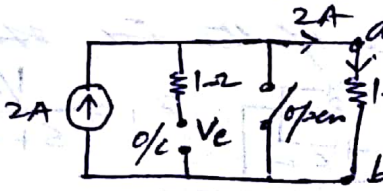
(ii) Given Z-parameters are

$$Z = \begin{bmatrix} 0.9 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$$

$$Y = Z^{-1} = \frac{1}{0.9\times0.6 - 0.2\times0.2}\begin{bmatrix} 0.6 & -0.2 \\ -0.2 & 0.9 \end{bmatrix}$$

$$\Rightarrow \boxed{Y_{22} = \frac{0.9}{0.5} = 1.8\ \mho}$$

(iii) At $t(0^-)$ ckt will be in steady state i.e. Capacitor will behave as open ckt. So the current flow through terminal ab



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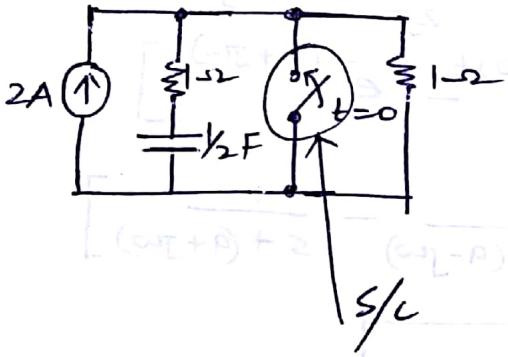
Thus voltage across capacitor will be same as the voltage across terminal 'ab'

i.e $V_c(0^-) = V_{ab} = 2 \times 1 = 2$ Volt.

At $t(0^+)$ When switch is closed

Since, $V_c(0^-) = V_c(0^+) \Rightarrow \boxed{V_c(0^+) = 2\,V}$

At $t(0^+)$ ckt becomes as below.



All the charges accumulated on capacitor will be disappeared through short circuited path.

Thus Voltage on capacitor at $\boxed{V_c(0^{\infty}) = 0V}$

(iv) From Foster-I form

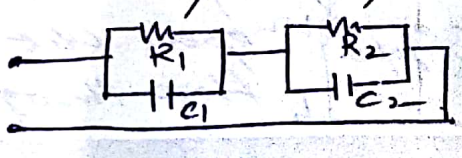
We can write $Z(s) = \dfrac{s+2}{(s+1)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$.

$\Rightarrow A = (s+1)\,Z(s)\Big|_{s=-1}$ and $B = (s+3)\,Z(s)\Big|_{s=-3}$

$\Rightarrow \boxed{A = \frac{1}{2} \quad ; \quad B = \frac{1}{2}}$

$\Rightarrow Z(s) = \dfrac{1/2}{s+1} + \dfrac{1/2}{s+3} = Z_1 + Z_2$

The above n/w is of the for standard form as



$Z_1 = \dfrac{1/C_1}{s + \frac{1}{R_1 C_1}}$ & $Z_2 = \dfrac{1/C_2}{s + 1/R_2 C_2}$

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(v). For checking stability check whether all the roots of the polynomials are L.H.S of s-plane.

The given polynomial is

$H(s) = s^3 + 3s^2 + 4s + 3$.

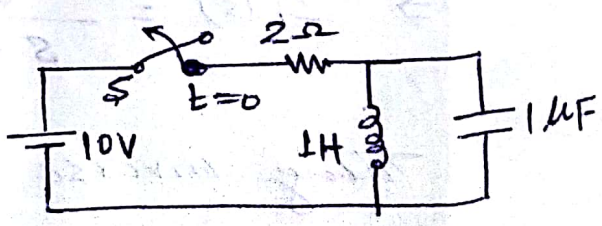
We can use R-Hurwitz theorem.

s^3	1	4
s^2	3	3
s^1	3	0
s^0	3	0

Since all the element of first column is +ve, So, all the roots of the polynomial lies on the left side of the s-plane.

Thus the given polynomial is **Stable**.

(02) At steady state (With switch 'S' is closed).



inductor behaves as a short ckt and capacitor behaves as a open ckt.

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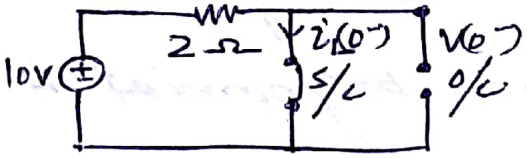
Case I: When Switch 'S' is Closed for a long time the ckt will be as given below.

$\Rightarrow i_L(0^-) = \frac{10}{2}$

$= 5$ Amp.



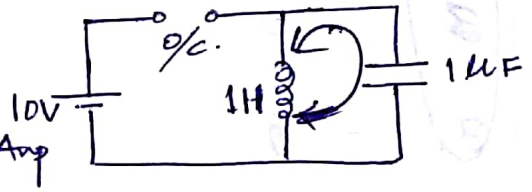
$V_c(0^-) = 0$ (as inductor behaves as a short circuit)

Case II: When Switch 'S' is opened at $t=0$

Circuit will be as given below.

For inductor

$i_L(0^+) = i_L(0^-) = 5$ Amp



Applying KVL in the LC loop.

$$\frac{L\, di_L(t)}{dt} + \frac{1}{C}\int_0^t i_L(t)\, dt = 0$$

$$= 1.\ \frac{di_L(t)}{dt} + \frac{1}{1\times 10^{-6}}\int_0^t i_L(t)\, dt = 0$$

$$= s\, I_L(s) - i_L(0^+) + 10^6\, \frac{I_L(s)}{s} = 0$$

$$\Rightarrow I_L(s) = \frac{5}{s + \frac{10^6}{s}} = \frac{5s}{s^2 + 10^6}$$

Taking inverse laplace transform

$$\Rightarrow \boxed{i^0(t) = 5\cos 1000\, t}$$

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(03) The given network is as shown.

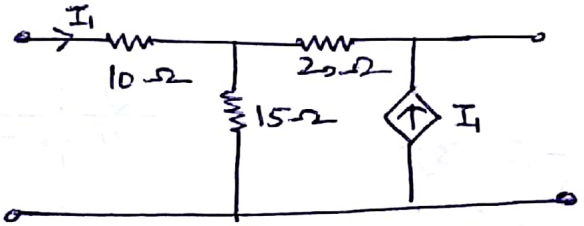
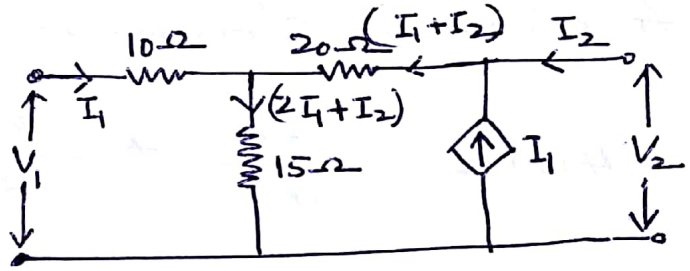


Fig. -I

Redrawing the n/w of Fig-I, it may be shown as below.



The loop equation becomes as follows

$$V_1 = 10I_1 + 15(2I_1 + I_2) \Rightarrow V_1 = 40I_1 + 15I_2 \quad \text{---(1)}$$

and $V_2 = 20(I_1 + I_2) + 15(2I_1 + I_2)$

or $V_2 = 50I_1 + 35I_2$ ---(2)

Therefore, from equation (1) & (2)

$Z_{11} = 40\Omega$; $Z_{21} = 50\Omega$

$Z_{12} = 15\Omega$; $Z_{22} = 35\Omega$

(04) The given function $H(s) = \dfrac{s(s+1)}{(s+4)(s^2+6s+18)}$

Writing in the factored form.

$$H(s) = \frac{s(s+1)}{(s+4)(s+3+3j)(s+3-j3)}$$

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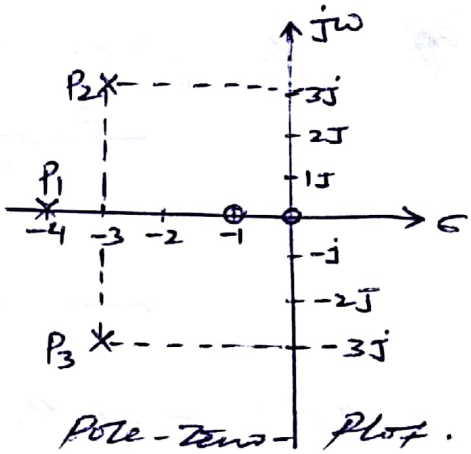
There are two zeros at $z_1 = 0$, and $z_2 = -1$

Poles are

$p_1 = -4$, $p_2 = -3 - 3j$

and $p_3 = -3 + 3j$



Pole-Zero Plot.

Figure-4

The pole-zero plot of the function is shown in figure-I.

$$H(s) = \frac{K_1}{s+4} + \frac{K_2}{s+3+j3} + \frac{K_3}{s+3-3j}$$

For the pole $p_1 = -4$

$$K_1 = \frac{(p_1 - z_1)(p_1 - z_2)}{(p_1 - p_2)(p_1 - p_3)} = 1.2$$

For p_2, $K_2 = \frac{(-1-8j)}{10}$

For p_3, $K_3 = \frac{-1+8j}{10}$

~~$K_2 = -3 - 3j$, $K_3 = -3 + 8j$~~

$$h(t) = \mathcal{L}^{-1}(H(s)) = K_1 e^{p_1 t} + K_2 e^{p_2 t} + K_3 e^{p_3 t}$$

$$= 1.2 e^{-4t} + \frac{1}{10}(-1-8j) e^{(-3-3j)t} + \frac{(-1+8j)}{10} \cdot e^{(-3+3j)t}$$

$$= 1.2 e^{-4t} + e^{-3t}\left[\frac{1}{10}\left(e^{3jt} + e^{-3jt}\right) + \frac{8}{10} j \left(e^{3jt} - e^{-3jt}\right)\right]$$

$$= 1.2 e^{-4t} + e^{3t}\left[-\frac{1}{10}(2\cos 3t) - \frac{8}{10}(2 \sin 3t)\right]$$

$$\boxed{h(t) = 1.2 e^{-4t} - \frac{1}{5} e^{-3t}\left[\cos 3t + 8 \sin 3t\right]}$$

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(05) The complete incidence matrix may be given as

$A_c =$

Nodes ↓ \ Branches →	1	2	3	4	5	6
a	0	1	0	−1	−1	1
b	0	−1	1	0	1	0
c	1	0	−1	0	0	0
d	−1	0	0	1	0	−1

It given that

$a_1 = 0,\ a_2 = 1,\ a_3 = 0,\ a_4 = -1,\ a_5 = -1,\ a_6 = 1$

i.e branches 2, 4, 5, and 6 are connected (incidence) on node a.

Branches 2 & 6 are away from node "a" since it's +ve.

Branches 4 & 5 are directed towards nodes 'a' since it's −ve.

Similarly, for other nodes and branches, applying similar reasoning, we can draw the respective branches that give the desired graph as shown in fig-5(a)

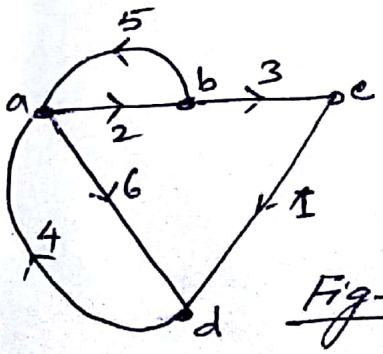

Fig-5(a) Oriented graph

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Select a tree of twigs (1, 2, 3) as shown in figure. 5(b)

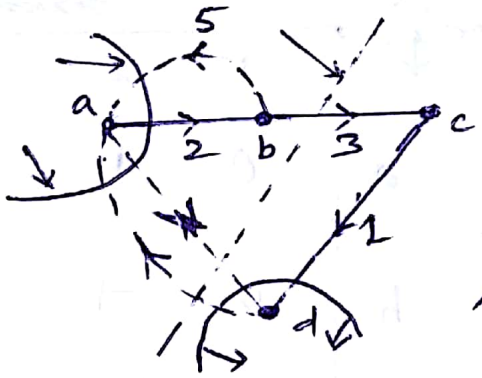

Figure: 5: (b)

Cut-set of the tree of twigs (1, 2, 3)

f-cut set matrix is given by

Q-f =

Cut-sets ↓ \ Branches →	1	2	3	4	5	6
1	1	0	0	-1	0	1
2	0	1	0	-1	-1	1
3	0	0	1	-1	0	1

END

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