1. (a) (iv) right angled triangle with equal sides

(b) (i) Reynolds' number

(c) (ii) equal to 1.0

(d) (ii) twice the depth of flow

(e) (i) minimum

(f) (ii) Turbulent boundary layer

(g) (iv) Coincides with the free surface

2. The velocity distribution in the boundary layer is given by

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

Calculate:

(a) Displacement Thickness **(b) Momentum thickness**

(c) Energy Thickness **(d) Shape factor**

Solution:

We have, Velocity distribution $\frac{u}{U} = (y/\delta)^{1/7}$

ⓐ Displacement thickness (δ^*)

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left[1 - (y/\delta)^{1/7}\right] dy$$

$$= \left\{ y - \left(\frac{1}{\delta^{1/7}}\right)\left[\frac{y^{(1/7+1)}}{(1/7+1)}\right]\right\}_0^\delta \qquad \left\{\because \int x^n dx = \frac{x^{n+1}}{n+1}\right\}$$

$$\Rightarrow \delta^* = \delta - \frac{1}{\delta^{1/7}} \frac{(\delta^{8/7})}{8/7}$$

$$= \delta - \frac{7}{8}\delta$$

$$\Rightarrow \boxed{\delta^* = \frac{\delta}{8}} \quad \text{Ans}$$

(b) Momentum Thickness (θ)

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7}\left[1 - (y/\delta)^{1/7}\right] dy$$

$$= \int_0^{\delta} \left\{\frac{y^{1/7}}{\delta^{1/7}} - \frac{y^{2/7}}{\delta^{2/7}}\right\} dy$$

$$= \left[\left(\frac{1}{\delta^{1/7}}\right) \frac{y^{8/7}}{(8/7)} - \frac{1}{(\delta^{2/7})}\left(\frac{y^{9/7}}{9/7}\right)\right]_0^{\delta}$$

$$= \frac{7}{8}\delta - \frac{7}{9}\delta$$

$$\Rightarrow \boxed{\theta = \frac{7}{72}\delta} \text{ Ans}$$

(c) Energy Thickness (δ_e):

$$\delta_e = \int_0^{\delta} \left[\frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)\right] dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{2/7}\right] dy = \int_0^{\delta} \left[(y/\delta)^{1/7} - (y/\delta)^{3/7}\right] dy$$

$$= \left[\frac{7}{8}\frac{(y)^{8/7}}{(\delta)^{1/7}} - \frac{7}{10} * \frac{y^{10/7}}{\delta^{3/7}}\right]_0^{\delta} = \left(\frac{7}{8}\delta - \frac{7}{10}\delta\right)$$

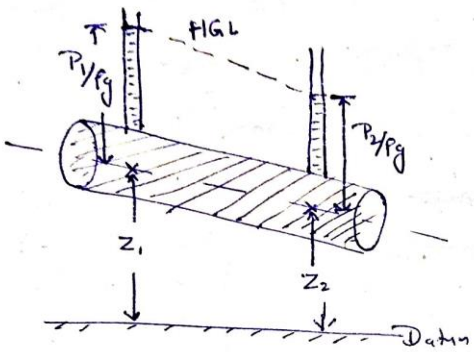
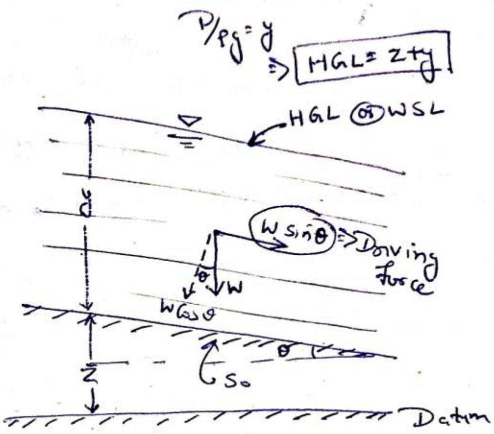
$$\Rightarrow \boxed{\delta_e = \frac{7}{40}\delta} \text{ Ans}$$

(d) Shape factor $= \frac{\delta^*}{\theta} = \frac{(\delta/8)}{(7\delta/72)} = \frac{72}{7*8} = \frac{9}{7} = 1.286$

$$\Rightarrow \boxed{\text{Shape factor} = 1.286} \text{ Ans}$$

3. (a) Explain how open channel flow is different from pipe flow.

Solution:

Pipe flow	Open Channel flow
	
$HGL = \frac{P}{\rho g} + z$	
(1) Flow takes place due to difference in pressure (or total energy diff.)	(1) Flow takes place due to gravity
(2) HGL ~~lie~~ lies above the top surface of water.	(2) HGL coincides with the top surface of water.
(3) Pipes are generally circular.	(3) Pipes may be of any shape such as Rectangular, triangular, circular, trapezoidal, etc.
(4) ~~Pressure Top Surface~~ Pressure is hydrostatic.	(4) Top surface of water should be subjected to atmospheric pressure
(5) Reynolds number is used for analysis. $Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V D}{\mu}$ $\Rightarrow \boxed{Re = \frac{\rho V D}{\mu}} = \frac{VD}{\nu}$	(5) Froude no. is used for analysis $\boxed{F_r = \sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{gL}}}$

NOTE: In open channel flow, channel can be open (or) close also, but the top surface of water should be subjected to atmospheric pressure

(b) Find the discharge in the following channels with a bed slope of 0.0006 and n = 0.016:

i. **Rectangular, B = 3.0 m, y_0 = 1.20 m**

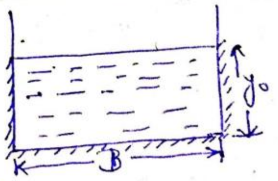
ii. **Trapezoidal, B = 3.0 m, m = 1.5 and y_0 = 1.10 m**

iii. **Triangular, m = 1.5, y_0 = 1.50 m.**

Solution:

Soln:- We have, $S_0 = 0.0006$, $n = 0.016$

(a) Rectangular



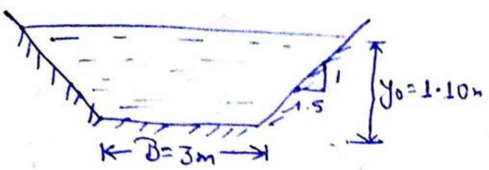
wetted area, $A = B * y_0 = 3 * 1.20 = 3.60\,m^2$

wetted Perimeter, $P = B + 2y_0 = 3 + 2*1.20 = 5.40\,m$

Hydraulic Radius, $R = \frac{A}{P} = \frac{3.60}{5.40} = 0.67\,m$

$\therefore\ Q = \frac{1}{n} A \cdot R^{2/3} S_0^{1/2} = \frac{1}{0.016} * (3.60)(0.67)^{2/3}(0.0006)^{1/2} = \boxed{4.22\,m^3/s}$ Ans

(b) Trapezoidal



$A = By_0 + my_0^2$

$= 3*1.1 + 1.5(1.1)^2 = 5.115\,m$

$P = B + 2y_0\sqrt{1+m^2}$

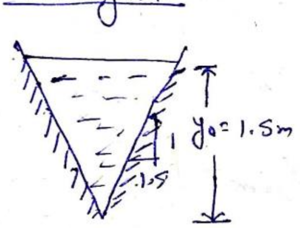
$= 3 + 2*1.1\sqrt{1+(1.5)^2} = 7.\,m$

$R = \frac{A}{P} = \frac{5.115}{7} = 0.73\,m$

$Q = \frac{1}{n} A \cdot R^{2/3} S_0^{1/2} = \frac{1}{0.016}(5.115)[0.73]^{2/3}(0.0006)^{1/2}$

$= \boxed{6.35\,m^3/sec}$ Ans

(c) Triangular



$A = my_0^2 = 1.5(1.5)^2 = 3.375\,m^2$

$P = 2y_0\sqrt{1+m^2} = 2*1.5\sqrt{1+(1.5)^2} = 5.41\,m$

$\therefore\ R = A/P = \frac{3.375}{5.41} = 0.62\,m$

$Q = \frac{1}{n} A \cdot R^{2/3} S_0^{1/2} = \frac{1}{0.016} * 3.375 * (0.62)^{2/3}(0.0006)^{1/2}$

$= \boxed{3.76\,m^3/sec}$

4. (a) What is a hydraulically efficient channel section? Explain

Solution:

Most Economical (or) Most Efficient Section of Channel:

A channel section is considered to be the most economical (or) most efficient when it can pass a maximum discharge for given c/s area, bed slope and roughness coefficient.

$$\cancel{Q=1} \quad Q = AV = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

Constant.

Thus, for maximum discharge, ⇒ R should be maximum

also, $R = A/P$

∴ R is maximum when P will be minimum

$$\therefore \quad Q \propto R^{2/3} \propto \frac{1}{P^{2/3}} \Rightarrow Q \propto \frac{1}{P}$$

∴ for maximum discharge, Perimeter should be minimum.

∴ Hence, most efficient channel section is most economical.

Explanation:

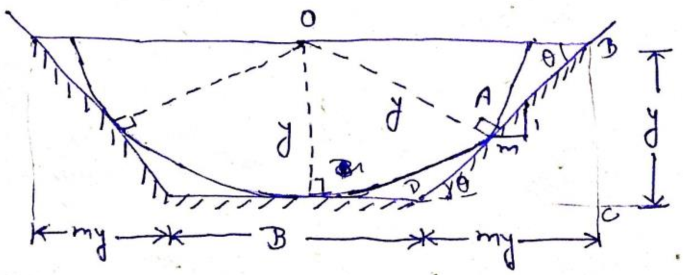
→ for a given area, if Perimeter is minimum, then, cost of lining of canal is ~~very less~~ minimum which is a major contributor in cost of construction of canal. Hence, channel is most economical.

→ for a given area, if Perimeter is minimum, then resistance from channel boundaries is minimum. Hence, velocity & discharge is maximum, channel is considered as most efficient

(b) Show that most economical and most efficient trapezoidal channel is half of a hexagon.

Solution:

Most economical (or) Most efficient Trapezoidal Channel Section:



Case 1 When Side Slope of Channel is Constant:

$$A = B*y + my^2 = (B+my)y$$

$$\Rightarrow \quad B = \frac{A}{y} - my$$

and,

$$P = B + 2y\sqrt{1+m^2} = \left(\frac{A}{y} - my\right) + 2y\sqrt{1+m^2}$$

for minimum P,

$$\frac{dP}{dy} = 0$$

$$\Rightarrow \quad \frac{-A}{y^2} - m + 2\sqrt{1+m^2} = 0$$

$$\Rightarrow -\frac{(B+my)y}{y^2} - m + 2\sqrt{1+m^2} = 0$$

$$\Rightarrow \quad \frac{B+my}{y} + m = 2\sqrt{1+m^2}$$

$$\boxed{\Rightarrow \quad \frac{B+2my}{2} = y\sqrt{1+m^2} \qquad \Rightarrow \quad \frac{T}{2} = \text{Length of one Side Slope}}$$ — first Criteria

$$R = A/P = \frac{(B+my)y}{B+2y\sqrt{1+m^2}} = \frac{(B+my)y}{B+(B+2my)} = y/2$$

$$\therefore \quad \boxed{R = y/2}$$ — 2nd Criteria.

In $\triangle OAB$

$$\sin\theta = \frac{OA}{OB} = \frac{OA}{(T/2)} \quad -(i)$$

In $\triangle BCD$

$$\sin\theta = \frac{BC}{BD} = \frac{y}{y\sqrt{1+m^2}} \quad -(ii)$$

equating (i) & (ii), we get

$$\frac{OA}{T/2} = \frac{y}{y\sqrt{1+m^2}} \Rightarrow \boxed{OA = y}$$ — 3rd Criteria

By considering O as a centre & depth of flow as a radius. If a circle is drawn then it touches channel bottom & sides tangentially only.

NOTE: In above three criteria, since m value is fixed therefore, channel will be efficient (or) economical only..
We have to find that value of m for which Channel is most efficient & most economical.

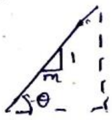
Case 2: When side slope of the channel varies:

$$P = B + 2y\sqrt{1+m^2}$$

$$\Rightarrow P = \left(\frac{A}{y} - my\right) + 2y\sqrt{1+m^2}$$

For best side slope $\Rightarrow \frac{dP}{dm} = 0$

$$\Rightarrow 0 - y + \frac{2y}{2\sqrt{1+m^2}} * 2m = 0 \Rightarrow \frac{2my}{\sqrt{1+m^2}} = y$$



$$\Rightarrow 4m^2 = 1 + m^2 \Rightarrow 3m^2 = 1$$

$$\Rightarrow \boxed{m = \frac{1}{\sqrt{3}}} \Rightarrow \cot\theta = \frac{1}{\sqrt{3}}$$

$\tan\theta = 1/m$

$\Rightarrow \tan\theta = \frac{1}{(1/\sqrt{3})}$

$$\Rightarrow \boxed{\theta = 60°}$$ — 4th Criteria.

NOTE: If all the four criteria are satisfied, then only Channel is considered to be most efficient (or) most economical

Since, from 1st Criteria, & from 4th Criteria

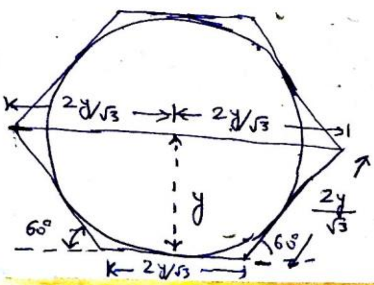
$$\frac{B + 2my}{2} = y\sqrt{1+m^2} \qquad m = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{B + 2y \times 1/\sqrt{3}}{2} = y\sqrt{1 + (1/\sqrt{3})^2}$$

$$\Rightarrow B + \frac{2y}{\sqrt{3}} = 2y\sqrt{1 + 1/3}$$

$$\Rightarrow B + \frac{2y}{\sqrt{3}} = \frac{4y}{\sqrt{3}} \Rightarrow B = \frac{2y}{\sqrt{3}}$$

length of one side slope $= y\sqrt{1+m^2} = y\sqrt{1 + 1/3} = \frac{2y}{\sqrt{3}}$



Most economical trapezoidal Channel is half of hexagon.

5. (a) Define Specific energy and Specific force.

Solution:

Specific Energy:

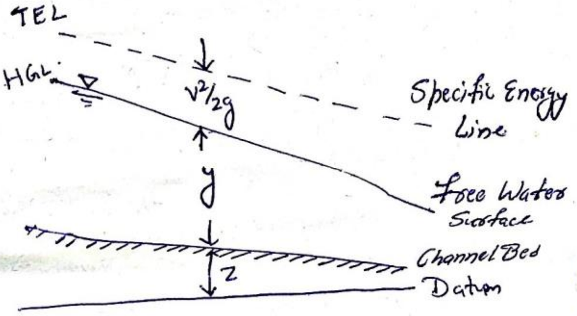
Here,

Total Energy, $TE = z + y + \frac{v^2}{2g}$

where, $z \Rightarrow$ Height of channel bed above datum

$y \Rightarrow$ depth of flow

$v \Rightarrow$ mean velocity of flow



→ If the channel bed is considered as datum, then total energy per unit weight of liquid will be

$$\boxed{E = y + \frac{v^2}{2g}} \Rightarrow \text{Specific Energy.}$$

→ Hence, Specific energy of a flowing liquid is defined as total energy per unit weight above the channel bed. It is the sum of Potential energy & Kinetic energy

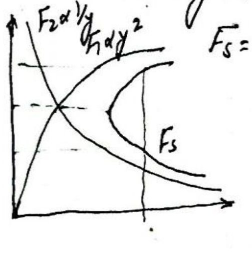
NOTE: ① Specific force ⇒ non-gravitational force per unit mass

② Sum of pressure force & momentum flux per unit weight.

③ $$\boxed{F_s = A\bar{z} + \frac{Q^2}{A \cdot g}}$$ $\bar{z} \Rightarrow$ Distance of c.g of section ~~from~~ top surface.

④ This equation is generally used in Rapidly Varied flow (Hydraulic Jump)

for Rectangular Channel.



$$F_s = A\bar{z} + \frac{Q^2}{A \cdot g} = (By)\frac{y}{2} + \frac{(qB)^2}{(By)g} = \frac{By^2}{2} + \frac{q^2 B}{g \cdot y}$$

$F_1 \propto y^2$

$F_2 \propto 1/y$

(b) A rectangular channel is 4.0 m wide and carries a discharge of 20 m^3/s at a depth of 2.0 m. At a certain section it is proposed to build a hump. Calculate the water surface elevations at upstream of the hump and over the hump if the hump height is (a) 0.33 m and (b) 0.20 m. (Assume no loss of energy at the hump.)

Solution:

Solution: Given, Rectangular Channel

$B = 4m$

$Q = 20 m^3/sec \Rightarrow q = Q/B = \frac{20}{4} = 5 m^3/s/m$

$y = 2m = y_1$

Now,

Critical depth, $y_c = (q^2/g)^{1/3} = \left(\frac{5^2}{9.81}\right)^{1/3} = \underline{1.366m}$

$E_c = \frac{3}{2} y_c = \frac{3}{2}(1.366) = \underline{2.05m}$

Maximum rise in bed (max^m height of hump) for which flow is possible (ΔZ_m)

$\Delta Z_m = E_1 - E_c$

$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2g y_1^2} = 2 + \frac{(5)^2}{2 * g * (2)^2} = 2.319m$

$\Rightarrow \Delta Z_m = 2.319 - 2.05 = \boxed{0.269m}$

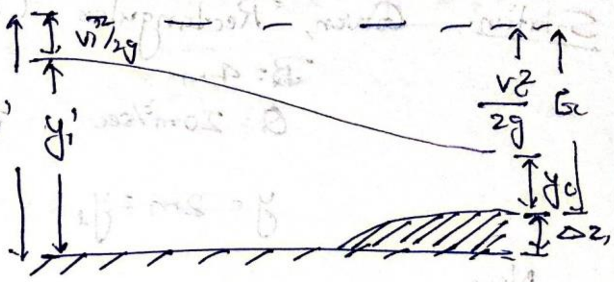
We have, (i) hump height $\underline{\Delta Z_1 = 0.33m} > \Delta Z_m$

thus, flow is not possible

It will be a case of Choking

thus, depth @ u/s has to be increased for flow to be possible

$E_1' = E_c + \Delta z_1$



$E_1' = E_c + \Delta z_1$

$(\Delta z_1 > \Delta z_m)$

$$\Rightarrow (y_1') + \frac{q^2}{2g(y_1')^2} = E_c + \Delta z_1$$

$$\Rightarrow (y_1') + \frac{5^2}{2(g)(y_1')^2} = 2.05 + 0.33$$

$$\Rightarrow (y_1')^3 - 2.38(y_1')^2 + 1.274 = 0$$

Solving, we get

$y_1' = 2.088\text{m} > y_c \Rightarrow \boxed{y_1' = 2.088\text{m}}$

$= -0.65\text{m} \Rightarrow -ve \Rightarrow \times$

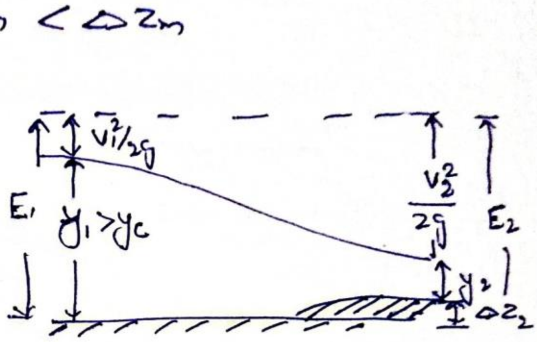
$0.94\text{m} < y_c \Rightarrow \times$

Thus Water Surface elevation (WSE)

(i) $\boxed{\begin{array}{l}\text{WSE @ u/s of hump} \Rightarrow y_1' = 2.088\text{m} \\ \text{WSE @ hump} \Rightarrow y_c = 1.366\text{m}\end{array}}$ Ans

(ii) hump height, $\Delta z_2 = 0.20m < \Delta z_m$

$\Rightarrow$ This is the normal case of flow



$\therefore \quad E_1 = E_2 + \Delta z_2$

$$\Rightarrow \quad y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2} + \Delta z_2$$

$$\Rightarrow \quad 2 + \frac{5^2}{2(9.81)(2)^2} = y_2 + \frac{(5)^2}{2g(y_2)^2} + 0.2$$

$$\Rightarrow \quad (y_2)^3 - 2.119(y_2)^2 + 1.274 = 0$$

Solving we get

$y_2 = 1.652m > y_c \Rightarrow$ $\boxed{y_2 = 1.652m}$

$-0.675m \Rightarrow -ve \Rightarrow \times$

$1.14m \Rightarrow < y_c \Rightarrow \times$

Thus (ii)
$$\boxed{\begin{array}{l} \text{WSE @ u/s of hump} \Rightarrow y_1 = 2m \\ \text{WSE @ hump} \Rightarrow y_2 = 1.652m \end{array}}$$

A_2

6. **(a) Define Gradually Varied Flow and Rapidly Varied Flow.**

Solution:

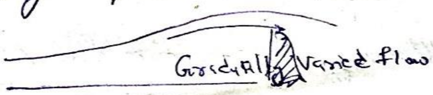
(2) <u>Uniform & Non-Uniform flow:</u>

If properties in a flow do not change w.r.t. space @ a particular instant ⇒ Uniform flow $\left.\frac{\partial R}{\partial s}\right|_t = 0$ R⇒ generally velocity ..

Otherwise; Non-uniform flow

→ <u>Gradually Varied Flow</u> ⇒ Depth of flow changes gradually along the length

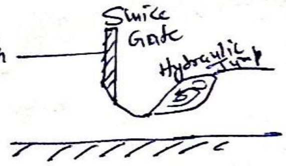
– friction plays important role here



→ <u>Rapidly Varied flow</u>

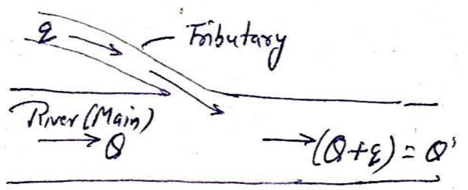
⇒ Depth of flow changes suddenly along the length

– friction is insignificant here



→ <u>Spatially Varied flow</u>

→ Change in depth occurs due to some addition (or) subtraction of water



NOTE: In GVF & RVF, there is no addition (or) subtraction of water

(b) Sketch the possible GVF profiles in the following serial arrangement of channels and control. The flow is from left to right:

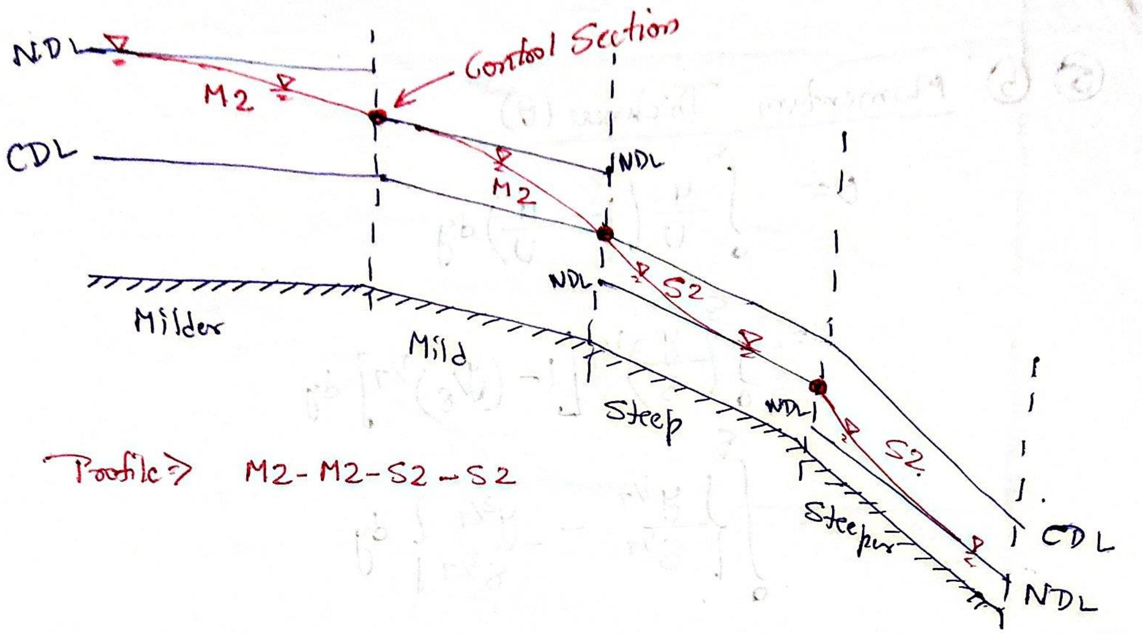
i. **milder - mild - steep – steeper**

ii. **steeper – steep – mild - milder**

iii. **mild – steeper – steep – sluice gate**

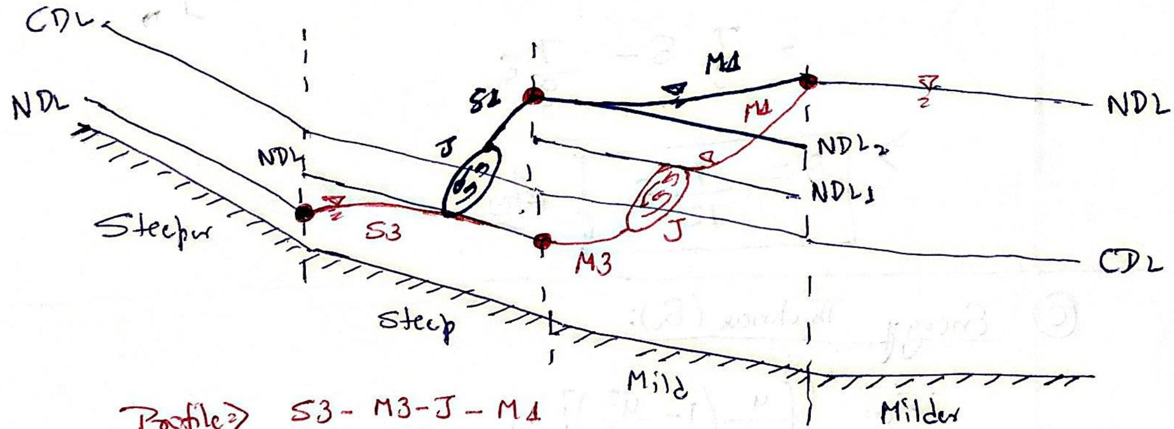
Solution:

(i) Milder - mild - Steep - steeper



Profile => M2 - M2 - S2 - S2

(ii) steeper - steep - mild - milder

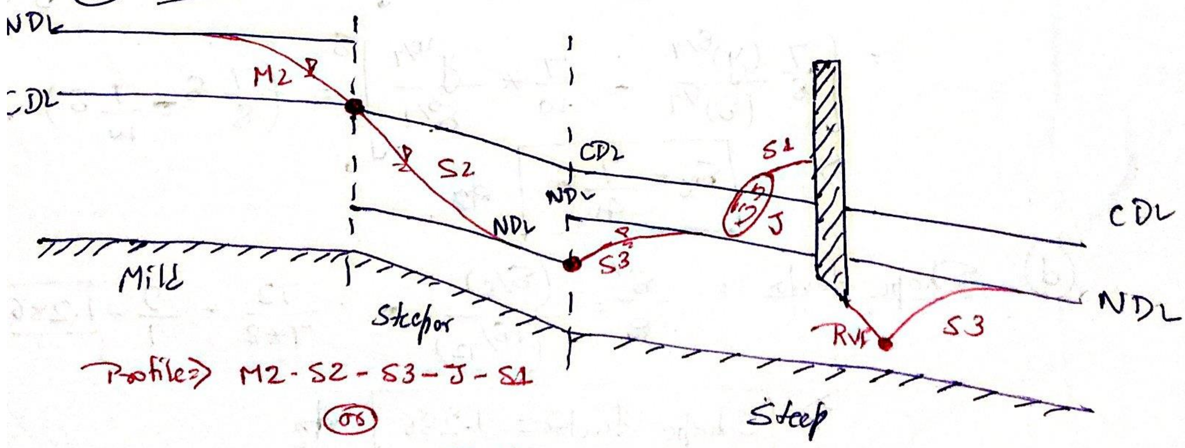


Profile => S3 - M3 - J - M1

or

S3 - J - S1 - M1

(iii) mild - Steeper - steep. sluice gate



Profile => M2 - S2 - S3 - J - S1

or

M2 - S2 - S3 - RVF - S3