Govt. of Bihar

MUZAFFARPUR INSTITUTE OF TECHNOLOGY, MUZAFFARPUR-842003

(Under the Department of Science & Technology Govt. of Bihar, Patna)

Mid-Semester Answer sheet, 2019

Subject Code- 031812 **Subject: Linear Control Theory**

Semester: 8^{th} **Department: ECE**

1. Obtain the root locus diagram for a unity feedback system with open loop transfer function. $G(s) = \dfrac{K}{s(s^2+6s+10)}$.

Answer:

(IES-EE-95-5M)

Sol: $G(s) = \dfrac{K}{s(s^2+6s+10)}$

No. of root locus branches =3(P>Z)

No. of asymptotes N = P − Z=3

$$\text{Angle of Asymptotes} = \frac{(2l+1)180^0}{P-Z} \quad l = 0,1,2$$

$$= 60^0, 180^0, 300^0$$

Centroid

$$\sigma = \frac{\left(\sum \text{real part poles} - \sum \text{real part of zeros}\right) \text{of } G(s)H(s)}{P-Z}$$

$$= \frac{0-3-3-0}{3} = -\frac{6}{3} = -2$$

Break away point

$$CE = 1 + KG_1(s)H_1(s) = 0$$

$$K = \frac{-1}{G_1(s)H_1(s)}$$

Roots of $\frac{dK}{ds} = 0$, which are on the root locus will be valid break away/Break in points.

$$G_1(s)H_1(s) = \frac{1}{s(s^2+6s+10)}$$

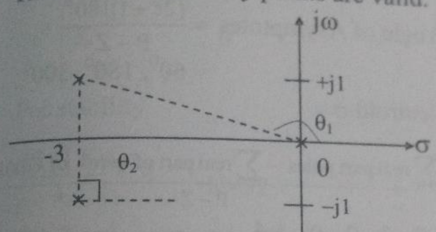
$$\frac{dK}{ds} = \frac{d}{ds}\left(\frac{-1}{G_1(s)H_1(s)}\right) = 0$$

$$\frac{d}{ds}\left(-s(s^2+6s+10)\right) = 0$$

$$3s^2 + 12s + 10 = 0$$

$s = -1.18$ and -2.81 rad/sec.

These two break away points are valid.



Angle of departure

$$\theta_1 = 180^0 - \tan^{-1}\left(\frac{1}{3}\right) = 161.6^0$$

$$\theta_2 = 90^0$$

Angle of departure $= 180 - (\theta_1 + \theta_2)$

$$= 180^0 - (161.6 + 90^0)$$

$$= -72^0. \text{ (At pole } -3+j1)$$

Angle of departure at pole $-3-j1$ is 72^0

Intersection of root locus with imaginary axi

CE is $s^3 + 6s^2 + 10 + K = 0$

Routh tabulation:

s^3	1	10
s^2	6	K
s^1	$\frac{60-K}{6}$	0
s^0	K	0

$\frac{60-K}{6} = 0$ Gives $K = 60$

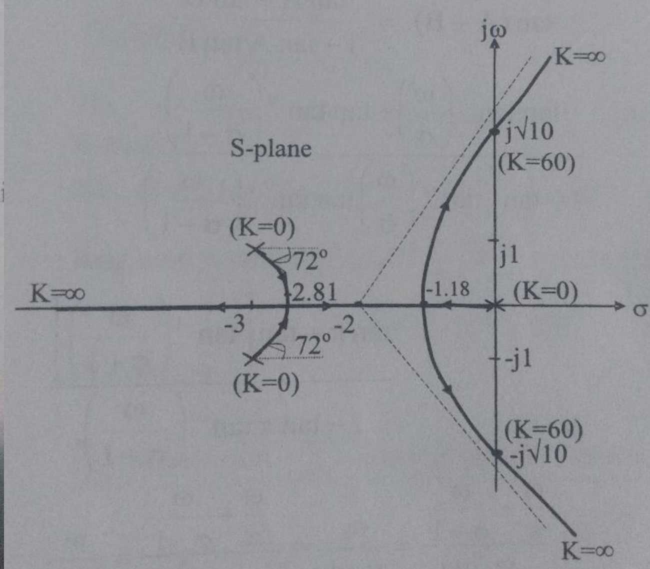
Auxiliary equation $= 6s^2 + K = 0$

$6s^2 + K = 0$ $(K=60)$

$6s^2 = -60,$ $s = \pm j\sqrt{10}$ Point of intersection

$\omega_n = \sqrt{10}$ rad/sec.

The RLD is shown in figure below



2. Sketch the root locus diagram for the closed loop system having a loop transfer function is given by

$$G(s)H(s) = \frac{K(s+2)}{s(s+1)}$$

Find the root locus as K is varied from 0 to ∞.

Answer:

$P = 2$ No of open loop poles

$Z = 1$ No of open loop zeros

No. of branches of RLD=2(2nd order system)

No.of Asymptotes $N = P - Z = 1$

$$\text{Angle of Asymptote} = \frac{(2\ell+1)180^0}{P-Z} \qquad l=0$$

$$= \frac{(2(0)+1)180^0}{1} = 180^0$$

Here, only one asymptote is present, hence centroid is not required.

Break Points

CE is $1 + KG_1(s)H_1(s) = 0$

$$G_1(s)H_1(s) = \frac{s+2}{s(s+1)}$$

$$\frac{d}{ds}G_1(s)H_1(s) = \frac{d}{ds}\left(\frac{s+2}{s(s+1)}\right) = 0$$

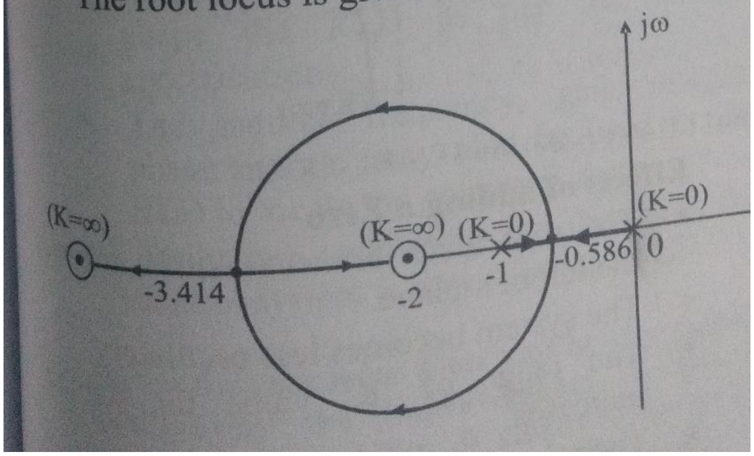
gives the break in / away point

After differentiating $s^2+4s+2=0$

roots are $s = -0.586$ and $s = -3.414$.

Break in/away Points are -0.586 and −3.414.

The root locus is given below



3. Draw the root locus of the system showing all the relevant points for open loop transfer function of the system given by $G(s) = \dfrac{K}{s(s^2+4s+8)}$.

Answer:

Sol. OLTF $G(s)=\frac{K}{s(s^2+4s+8)}$

No. of root locus branches=3(P>Z)

No. of Asymptotes = N = P-Z = 3-0= 3

Angle of Asymptotes $=\frac{(2\ell+1)180^0}{P-Z}$

$= 60^0, 180^0, 300^0$

Centroid σ =

$$\frac{\left(\sum \text{real part poles} - \sum \text{real part of zeros}\right) \text{of } G(s)H(s)}{P-Z}$$

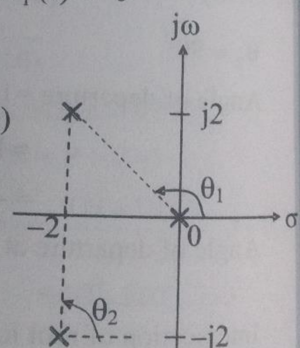
$$=\frac{0-2-2-0}{3}=\frac{-4}{3}=-1.33$$

Break away point

CE is $1+KG_1(s)H_1(s) = 0$

$$K=\frac{-1}{G_1(s)H_1(s)}$$

$$=-s(s^2+4s+8)$$



$$\frac{dK}{ds}=\frac{d}{ds}\left(\frac{-1}{G_1(s)H_1(s)}\right)=0$$

$$\frac{d}{ds}\left(-s(s^2+4s+8)\right)=0$$

$3s^2+8s+8=0$

$s=-1.3\pm j0.94,$

These points are not valid breakaway/in points.

Angle of departure

$\theta_1=135^0$ $\qquad \theta_2=90^0$

Angle of Departure $=180^0-(\theta_1+\theta_2)$

$=180^0-(135+90^0)$

$=-45^0$ at s=-2+j2

Hence angle of departure at s = -2 −j2 is +45°

Intersection of the RLD with the imaginary axis

CE = $s^3+4s^2+8s+K = 0$

Routh 's tabulation

s^3	1	8
s^2	4	K
s^1	$\frac{32-K}{4}$	0
s^0	K	0

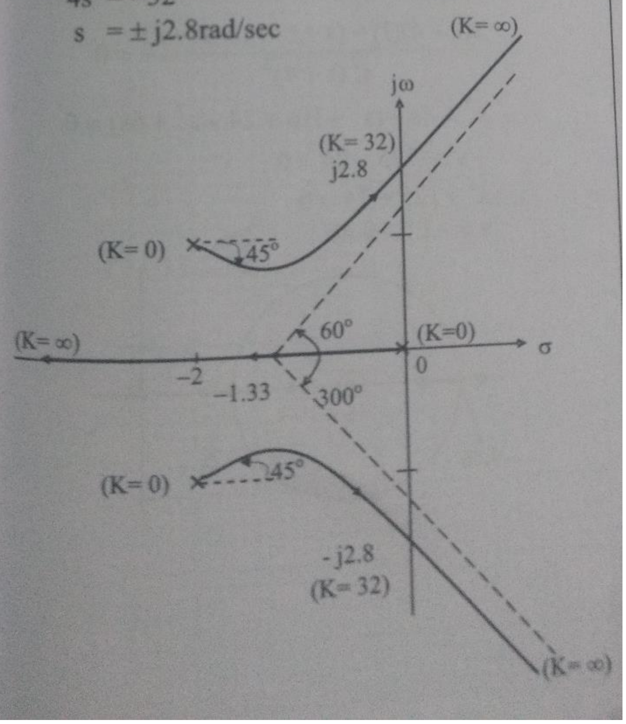
For stability

$\frac{32-K}{4}=0$, K=32

Auxiliary equation is $4s^2+K=0$ (K=32)

$4s^2=-32$

$s = \pm j2.8$rad/sec



4. Sketch the Nyquist Plot for the control system whose loop transfer function is given by $G(s)H(s)=\frac{1}{s(1+0.2s)(1+0.05s)}$.

Answer:

Sol: $G(s)\,H(s) = \dfrac{1}{s(1+0.2s)(1+0.05s)}$

$$= \frac{K}{s(1+s\tau_1)(1+s\tau_2)}$$

$K=1,\ \tau_1=0.2,\ \tau_2=0.05,$

$$\omega_{pc} = \frac{1}{\sqrt{\tau_1\tau_2}} = \sqrt{\frac{1}{0.2\times0.05}} = 10$$

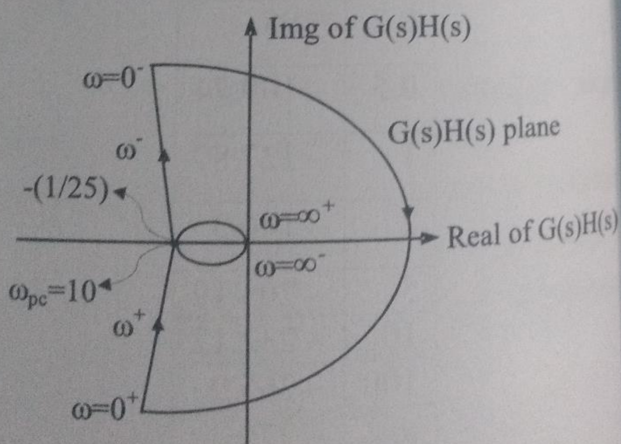
$\therefore$ Phase cross over frequency $\omega_{pc} = 10$

$$\text{Gain Margin} = 20\log\frac{1}{K}\left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}\right)$$

$$= 20\log 25$$

$$= 28\text{dB, System is stable}$$

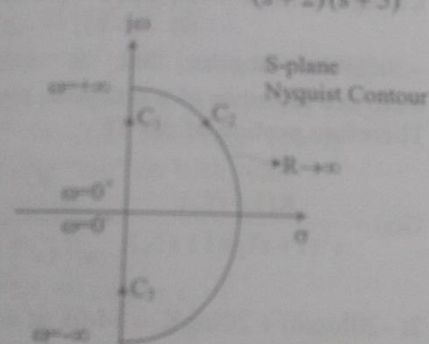
Nyquist plot is shown in figure.



5 The open loop transfer function of a unity feedback control system is given by the expression $G(s) = \dfrac{K}{(s+2)(s+5)}$. Draw the Nyquist plot of the closed loop system and comment upon the stability of the system.

Answer:

Sol: Given that $G(s)H(s) = \dfrac{K}{(s+2)(s+5)}$



Nyquist plot is the mapping of Nyquist contour(S-plane) into G(s)H(s) plane.

The Nyquist contour in the s-plane enclosing the entire right half of S-plane is shown below.

The Nyquist Contour has three sections C_1, C_2 and C_3. These sections are mapped into G(s)H(s) plane

Mapping of section C_1: It is the positive imaginary axis, therefore sub $s = j\omega$, $(0 \le \omega \le \infty)$ in the TF G(s)H(s), which gives the polar plot

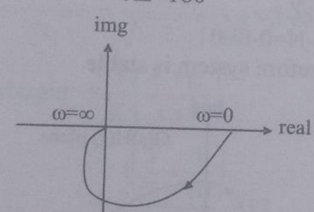
$$G(s)H(s) = \frac{K}{(s+2)(s+5)}$$

$$G(s)H(s) = \frac{K}{(j\omega+2)(j\omega+5)}$$

$$G(j\omega)H(j\omega) = \frac{K}{\sqrt{(\omega^2+4)(\omega^2+25)}} \angle -(\tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{5})$$

$\omega = 0$ $\quad K\angle -0^0$

$\omega = \infty$ $\quad 0\angle -180^0$

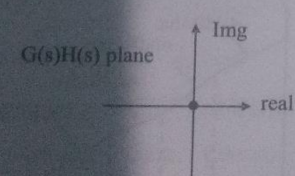


Mapping of section C_2: It is the radius 'R' semicircle, therefore sub $s = \lim_{R\to\infty} Re^{j\theta}$ (θ is from 90^0 to 0^0 to -90^0) in the TF G(s)H(s), which merges to the origin in G(s)H(s) plane.

$$G(s)H(s) = \frac{K}{(s+2)(s+5)}$$

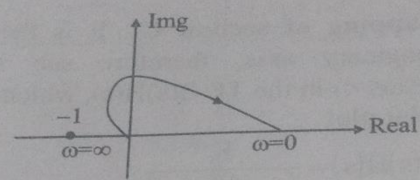
$$G(Re^{j\theta})H(Re^{j\theta}) = \frac{k}{(Re^{j\theta}+2)(Re^{j\theta}+5)} \approx 0$$

The plot is shown in figure above.



Mapping of section C_3: It is the negative imaginary axis, therefore sub $s = j\omega$,

$(-\infty \le \omega \le 0)$ in the TF G(s) H(s), which gives the mirror image of the polar plot and is symmetrical with respect to the real axis,



The plot is shown in figure.

Combining all the above four sections, the Nyquist plot of $G(s)H(s) = \frac{K}{(s+2)(s+5)}$ is shown in figure.

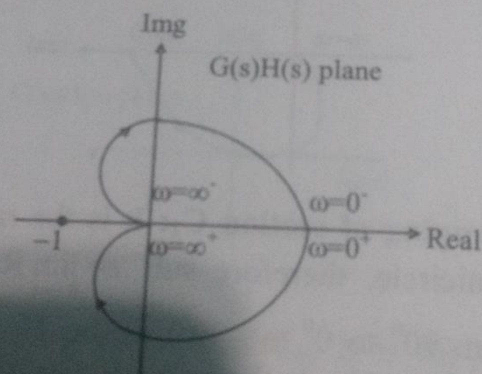
From the plot N=0

Given that P=0

N=P-Z

Z=P-N=0-0=0

Therefore system is stable



(a)