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(Under the department of Science & Technology, Bihar, Patna)

B.Tech 4th Semester Mid-Term Examination, 2019

MECHANICS OF SOLID, CIVIL ENGINEERING

MODEL ANSWER

1) Choose the correct option from the following. (Only one option is correct)

(i) Which one of the following statements is correct?

a) Shear force is the first derivative of bending moment.

b) Shear force is the first derivative of intensity of load.

c) Load intensity on a beam is the first derivative of bending moment.

d) Bending moment is the first derivative of shear force.

Ans (a) Shear force is the first derivative of bending moment.

(ii) Consider the following statements:

A simply-supported beam is subjected to a couple somewhere in the span. It would produce.

1. A rectangular SF diagram
2. Parabolic BM diagrams
3. Both +ve and –ve BMs which are maximum at the point of application of the couple.

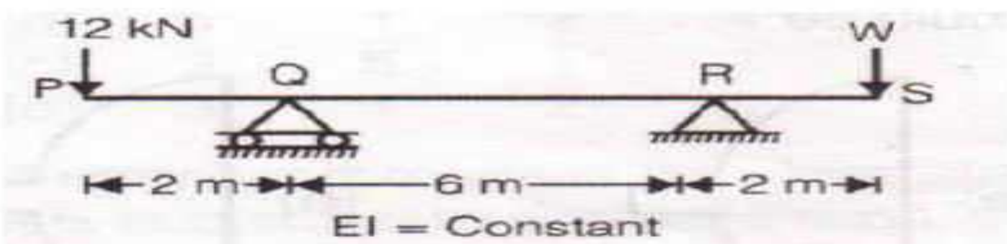
Which of these statements are correct?

a) 1,2 and 3 b) 1 and 2

c) 2 and 3 d) 1 and 3

Ans (d) 1 and 3

(iii) A loaded beam PQRS is shown in the given figure. The magnitude of reaction at R will be Zero if the Value of load is



a) 2 KN b) 2.5 KN c) 3 KN d) 6 KN

Ans (C) 3kN

Hint: Taking moment about Q

12 X 2= W X (6+2)

W= 24/8= 3 kN

(iv) A simply supported beam AB of span L carries two concentrated loads W each at Point L/3 from A and B. what is the SF in the middle one-third portion of the beam?

a) W/2 b) W c) 2W d) Zero

Ans (d) Zero

(v) The shear modulus (G), modulus of elasticity (E) and the Poisson's ratio (v) of a material are related as,

(A) $G = E/[2(1 + v)]$ (B) $E = G/[2(1+v)]$

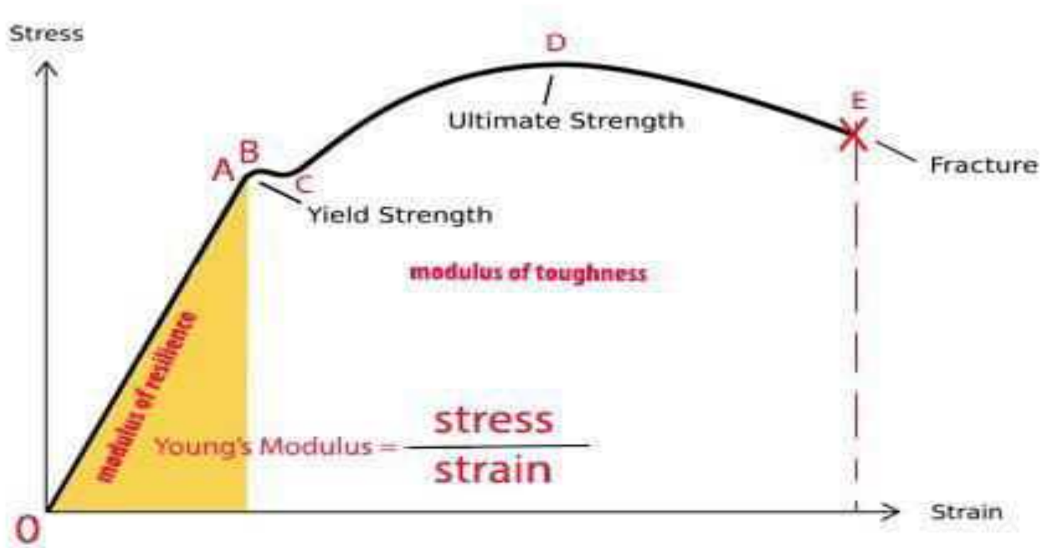
(C) $G = E/[2(1 - v)]$ (D) $G = E/[2(v - 1)]$

Ans (a) $G = E/[2(1 + v)]$

2) Draw the stress –strain curve for mild steel.

Solution-

Stress – Strain Curve for Mild Steel



If tensile force is applied to a steel bar it will have some extension. If the force is small the ratio of the stress and strain will remain proportional. And the graph will be a straight line (up to point A) . So the o to point A is the **limit of proportionality**.

If the force is considerably large the material will experience elastic deformation but the ratio of stress and strain will not be proportional. (point A to B). This is the elastic limit**.** Beyond that point the material will experience plastic deformation. The point where plastic deformations starts is the yield point which is show in the figure as point B. o B is the upper yield point. Resulting graph will not be straight line anymore. C is the lower yield point. D is the maximum ultimate stress. E is the breaking stress.

Some more terms associated with the stress strain graph

Hooke's law: Within the proportional limit strain is proportionate to stress.

Young's modulus of elasticity

Within the proportional limit, stress α strain, hence stress = E × strain

E is a proportionality constant known as Modulus of elasticity or Young's modulus of elasticity.

This constant of proportionality is called Young's modulus of elasticity and is given the symbol E.

E has the same unit as the unit of stress because strain is dimensionless. $E = \sigma / \varepsilon$ Pa

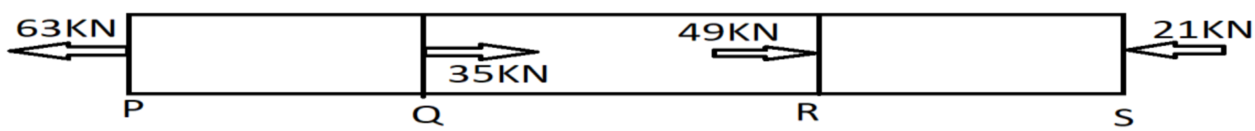
Another two terms are important in the stress strain graph of mild steel these are

Modulus of Resilience: It is the area under the curve which is marked by the yellow area. It is the energy absorbed at unit volume up to elastic limit.

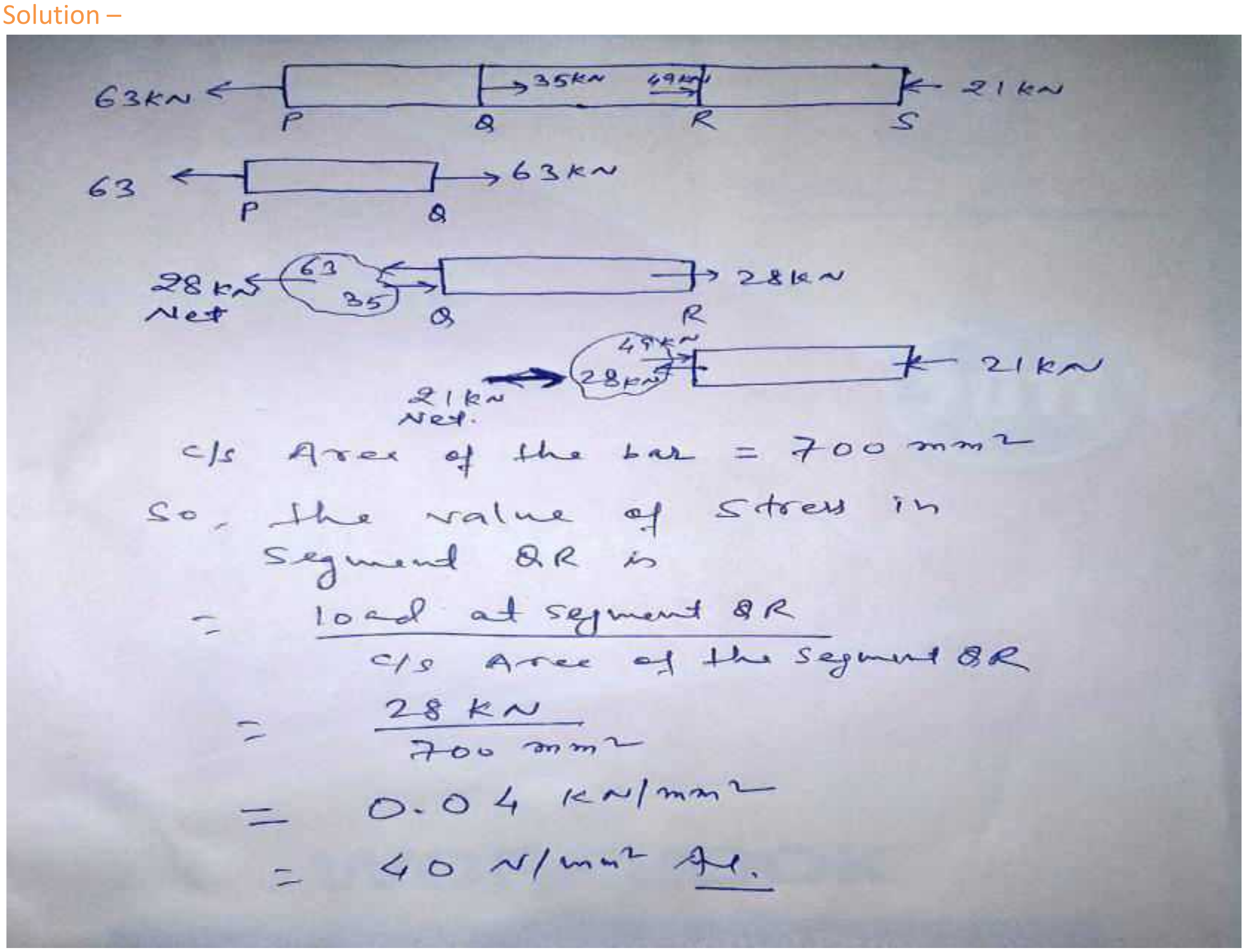
modulus of resilience = $1/2 \times \sigma \times \varepsilon = 0.5 \times (FL/AE)$

Modulus of toughness: It is the area of the whole curve (point 0-E). Energy absorbed at unit volume up to breaking point.

3) A bar having a cross sectional area of 700mm² is subjected to axial loads at the position indicated. The value of stress in segment QR is



Solution –



c/s Area of the bar = 700 mm²

So, the value of stress in segment QR is

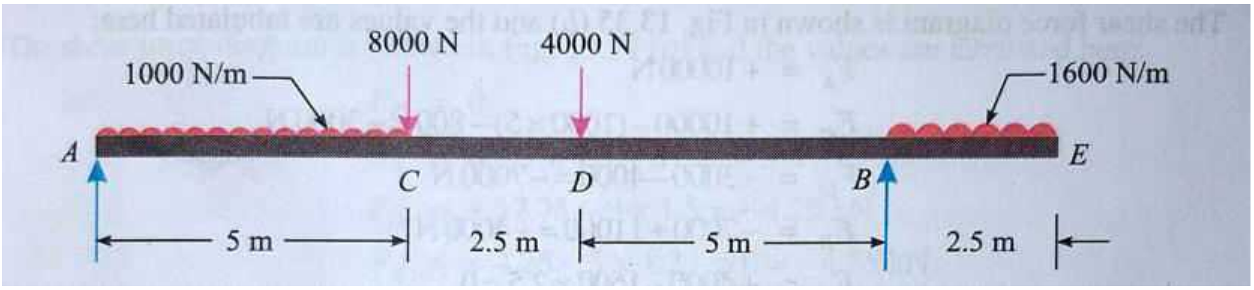
$= \dfrac{\text{load at segment QR}}{\text{c/s Area of the segment QR}}$

$= \dfrac{28 \text{ kN}}{700 \text{ mm}^2}$

$= 0.04 \text{ kN/mm}^2$

$= 40 \text{ N/mm}^2$ Ans.

4) Draw shear force and bending moment diagrams for the beam shown in figure indicate the numerical values at all important sections



Solution-

Given : Span $(l) = 15$ m ; Uniformly distributed load between A and B $(w_1) = 1000$

N/m ; Point load at C $(W_1) = 8000$ N ; Point load at D $(W_2) = 4000$ N and uniformly distributed load between B and E $(w_2) = 1600$ N/m.

First of all, let us find out the reactions R_A and R_B.

Taking moments about A and equating the same,

$$R_B \times 12.5 = (1600 \times 2.5) \times 13.75 + (4000 \times 7.5) + (8000 \times 5) + (1000 \times 5) \times 2.5$$

$$= 137500$$

$$R_B = \frac{137500}{12.6} = 110000 \text{ N}$$

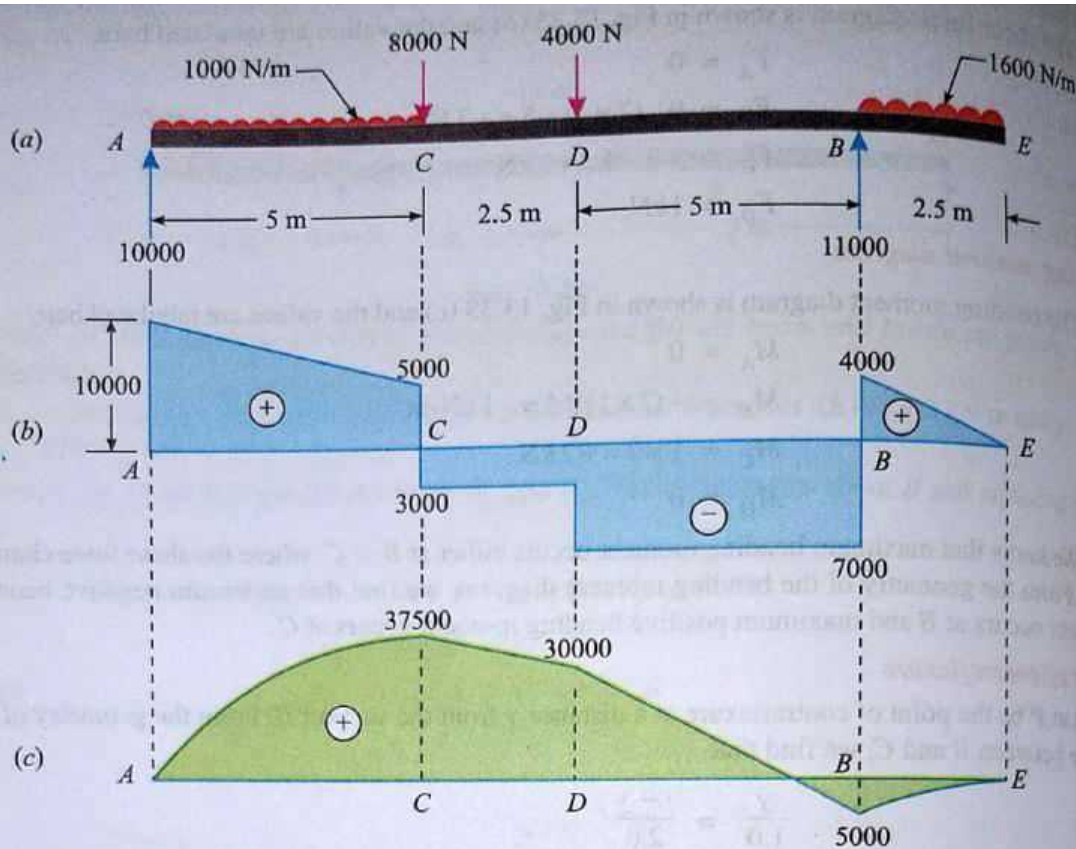


Fig. 13.35

and

$$R_A = (1000 \times 5 + 8000 + 4000 + 1600 \times 2.5) - 11000\,\text{N}$$
$$= 10000\,\text{N}$$

Shear force

The shear force diagram is shown in Fig. 13.35 (*b*) and the values are tabulated here:

$$F_A = +10000\,\text{N}$$
$$F_C = +10000 - (1000 \times 5) - 800 = -3000\,\text{N}$$
$$F_D = -3000 - 4000 = -7000\,\text{N}$$
$$F_B = -7000 + 11000 = +4000\,\text{N}$$
$$F_E = +4000 - 1600 \times 2.5 = 0$$

Bending moment

The bending moment diagram is shown in Fig. 13.35 (*c*), and the values are tabulated here:

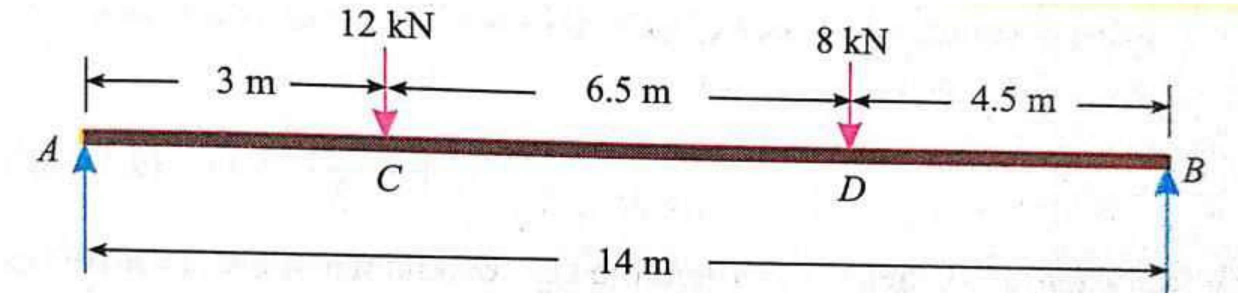
$$M_A = 0$$
$$M_C = (10000 \times 5) - (1000) \times (5 \times 2.5) = 37500\,\text{N-m}$$
$$M_D = (10000 \times 7.5) - (1000 \times 5 \times 5) - (8000 \times 2.5)\,\text{N-m}$$
$$= 30000\,\text{N-m}$$
$$M_B = -1600 \times 2.5 \times \frac{2.5}{2} = -5000\,\text{N-m}$$

Maximum bending moment

The maximum bending moment, positive or negative will occur at *C* or at *B* because the shear force changes sign at both these points. But from the bending moment diagram, we see that the maximum positive bending moment occurs at *C* and the maximum negative bending moment occurs at *B*.

5) A horizontal steel girder having uniform cross-section is 14 m long and is simply supported at its ends. It carries two concentrated loads as shown in figure. Calculate the deflection of the beam under the loads C and D. take E =200 GPa and I =160x10^6 mm^4

Solution-



Solution-

Given: Span (l) = 14 m = 14×10^3 mm ; Load at C (W_1) = 12 kN = 12×10^3 N ; Load at D (W_2) = 8 kN = 8×10^3 N ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and moment of inertia (I) = 160×10^6 mm^4.

Taking moments about A and equating the same,

$$R_B \times 14 = (12 \times 3) + (8 \times 9.5) = 112$$

$$\therefore \quad R_B = \frac{112}{14} = 8 \text{ kN} = 8 \times 10^3 \text{ N}$$

and

$$R_A = (12 + 8) - 8 = 12 \text{ kN} = 12 \times 10^3 \text{ N}$$

Now taking A as the origin and using Macaulay's method, the bending moment at any section X at a distance x from A,

$$EI\frac{d^2y}{dx^2} = -(12 \times 10^3)\,x + \;\vdots\; (12 \times 10^3) \times [x - (3 \times 10^3)] + (8 \times 10^3) \times [x - (9.5 \times 10^3)]$$

Integrating the above equation,

$$EI\frac{dy}{dx} = -(12\,t\,10^3)\frac{x^2}{2} + \;\vdots\; C_1 + (12 \times 10^3) \times \frac{[x - (3 \times 10^3)]^2}{2} + (8 \times 10^3) \times \frac{[x - (9.5 \times 10^3)]^2}{2}$$

$$= -(6\times10^3)\,x^2 + C_1 \quad + (6\times10^3)\times[x-(3\times10^3)]^2$$

$$+ (4\times10^3)\times[x-(9.5\times10^3)] \qquad ...(i)$$

Integrating the above equation once again,

$$EI\cdot y = -(6\times10^3)\times\frac{x^3}{3} + C_1\,x + C_2 + \quad (6\times10^3)\times\frac{[x-(3\times10^3)]^3}{3}$$

$$+ (4\times10^3)\times\frac{[x-(9.5\times10^3)]^3}{3}$$

$$= (2\times10^3)\,x^3 + C_1\,x + C^2 \quad + (2\times10^3)\,[x-(3\times10^3)]^3$$

$$+ \frac{4\times10^3}{3}\times(x-(9.5\times10^3)]^3 \qquad ...(ii)$$

We know that when $x = 0$, then $y = 0$. Therefore $C_2 = 0$. And when $x = (14\times10^3)$ mm, then $y = 0$. Therefore

$$0 = -(2\times10^3)\times(14\times10^3)^3 + C_1\times(14\times10^3)$$
$$+ (2\times10^3)\times[(14\times10^3)-(3\times10^3)]^3$$
$$+ \frac{4\times10^3}{3}\times[(14\times10^3)\;(9.5\times10^3)]^3$$
$$= -(5488\times10^{12}) + (14\times10^3)\,C_1 + (2662\times10^{12}) + 121.5\times10^{12}$$
$$= -(2704.5\times10^{12}) + (14\times10^3)\,C_1$$

$$\therefore \qquad C_1 = \frac{2704.5\times10^{12}}{14\times10^3} = 193.2\times10^9$$

Substituting the value of C_1 equal to 193.2×10^9 and $C_2 = 0$ in equation (*ii*),

$$EIy = -2\times10^3\,x^3 + 193.2\times10^9\,x \quad + 2\times10^3\,[x-(3\times10^3)]3$$

$$+ \frac{4\times10^3}{3}\times[x-(9.5\times10^3)]^3 \qquad ...(iii)$$

Now for deflection under the 12 kN load, substituting $x = 3$ m (or 3×10^3 mm) in equation (*iii*) up to the first dotted line only,

$$EIy_C = -2\times10^3\times(3\times10^3)^3 + 193.2\times10^9\times(3\times10^3)$$
$$= -(54\times10^{12}) + (579.6\times10^{12}) = 525.6\times10^{12}$$

$$\therefore \qquad y_C = \frac{525.6\times10^{12}}{EI} = \frac{525.6\times10^{12}}{(200\times10^3)\times(160\times10^6)} = 16.4\ \text{mm} \qquad \text{Ans.}$$

Similarly, for deflection under the 8 kN load, substituting $x = 9.5$ m (or 9.5×10^3 mm) in equation (*iii*) up to the second dotted line only,

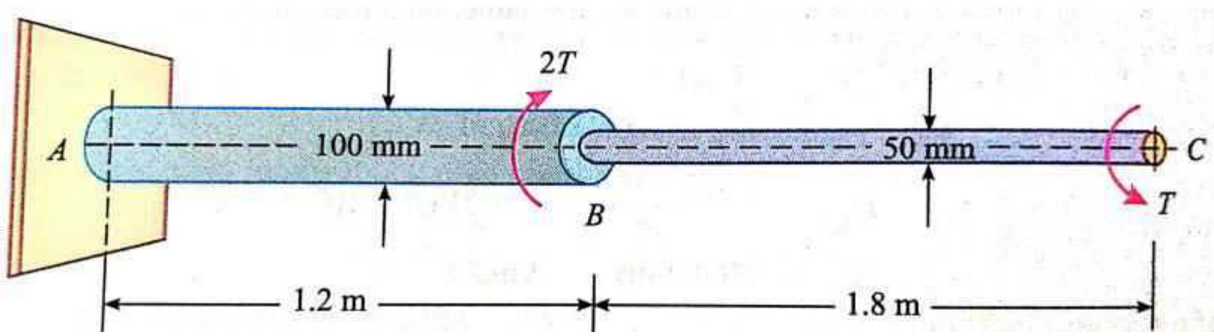
$$EI\, y_D = -2 \times 10^3 \times (9.5 \times 10^3)^3 + 193.2 \times 10^9 \times (9.5 \times 10^3) + 2 \times 10^3 \times [(9.5 \times 10^3) - (3 \times 10^3)]^3$$

$$= -(1714.75 \times 10^{12}) + (1835.4 \times 10^{12}) + (549.25 \times 10^{12})$$

$$= 669.9 \times 10^{12}$$

$$\therefore \quad y_D = \frac{669.9 \times 10^{12}}{EI} = \frac{669.9 \times 10^{12}}{(200 \times 10^3) \times (160 \times 10^6)} = \mathbf{20.9\ mm} \quad \textbf{Ans.}$$

6) The stepped steel shafts shown in figure is subjected to a torque(T) at the free end, and a torque (2T) in the opposite derection at the junction of the two sizes. What is the total angle of twist at the free end, if maximum shear stress in the shaft is limited to 70 MPa? Assume the modulus of rigidity to be 84 GPa.



Solution-

Given: Torque at $C = T$ (anticlockwise); Torque at $B = 2T$ (clockwise) ; Diameter of shaft AB (D_{AB}) = 100 mm ; Diameter of shaft BC (D_{BC}) = 50 mm ; Maximum shear stress (τ) = 70 MPa = 70 N/mm^2 and modulus of rigidity (C) = 84 GPa = 84×10^3 N/mm^2.

Since the torques at B and C are in opposite directions, therefore the effect of these two torques will be studied first independently, sum of the two twists (one in clockwise direction and the other in anticlockwise direction).

First of all, let us first find out the value of torque T at C. It may be noted that if the value of torque is obtained for the portion AB, it will induce more stress in the portion BC (because the portion BC is of less diameter). Therefore we shall calculate the torque for the portion BC (because it will not induce stress more than the permissible in the portion AB).

We know that the torque at C,

$$T = \frac{\pi}{16} \times \tau \times (D_{BC})^3 = \frac{\pi}{16} \times 70 \times (50)^3 = 1.718 \times 10^6 \text{ N-mm}$$

We also know that polar moment of inertia of the solid circular shaft AB,

$$J_{AB} = \frac{\pi}{32} \times (D_{AB})^4 = \frac{\pi}{32} \times (100)^4 = 9.82 \times 10^6 \text{ mm}^4$$

Similarly,

$$J_{BC} = \frac{\pi}{32} \times (D_{BC})^4 = \frac{\pi}{32} \times (50)^4 = 0.614 \times 10^6 \text{ mm}^4$$

$\therefore$ Angle of twist at C due to torque (T) at C,

$$\theta = \frac{T \cdot l}{J \cdot C} = \frac{T}{C}\left(\frac{l_{AB}}{J_{AB}} + \frac{l_{BC}}{J_{BC}}\right)$$

$$= \frac{1.718 \times 10^6}{84 \times 10^3}\left(\frac{1.2 \times 10^3}{9.82 \times 10^6} + \frac{1.8 \times 10^3}{0.614 \times 10^6}\right) \text{rad}$$

$$= 20.45 \times (30.54 \times 10^{-4}) = 0.0624 \text{ rad} \qquad ...(i)$$

Similarly, angle of twist at C due to torque ($2T$) at B,

$$\theta = \frac{2T}{C} \times \frac{l_{AB}}{J_{AB}} = \frac{2 \times (1.718 \times 10^6)}{84 \times 10^3} \times \frac{1.2 \times 10^3}{9.82 \times 10^6} \text{ rad}$$

$$= 40.9 \times (1.222 \times 10^{-4}) = 0.005 \text{ rad} \qquad ...(ii)$$

From the geometry of the shaft, we find that the twist at B (due to torque of $2T$ at B) will continue at C also. Since the directions of both the twists are opposite to each other, therefore net angle of twist at C

$$= 0.0624 - 0.005 = 0.0574 \text{ rad} = \mathbf{3.29°} \qquad \textbf{Ans.}$$