MUZAFFARPUR INSTITUTE OF TECHNOLOGY

B.Tech 6th Semester Mid-Sem Exam., 2019

DIGITAL IMAGE PROCESSING (ECE)

TIME: 2 Hrs **FULL MARKS: 20**

Ques. (1): Explain the following term **(any three)** **(2 x 3 = 6 marks)**

(a) Formula of Fourier Transform in two-dimensional space.

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)}$$

$$\text{for } u = 0,1,2,\ldots,M-1, v = 0,1,2,\ldots,N-1$$

(b) Image Negatives

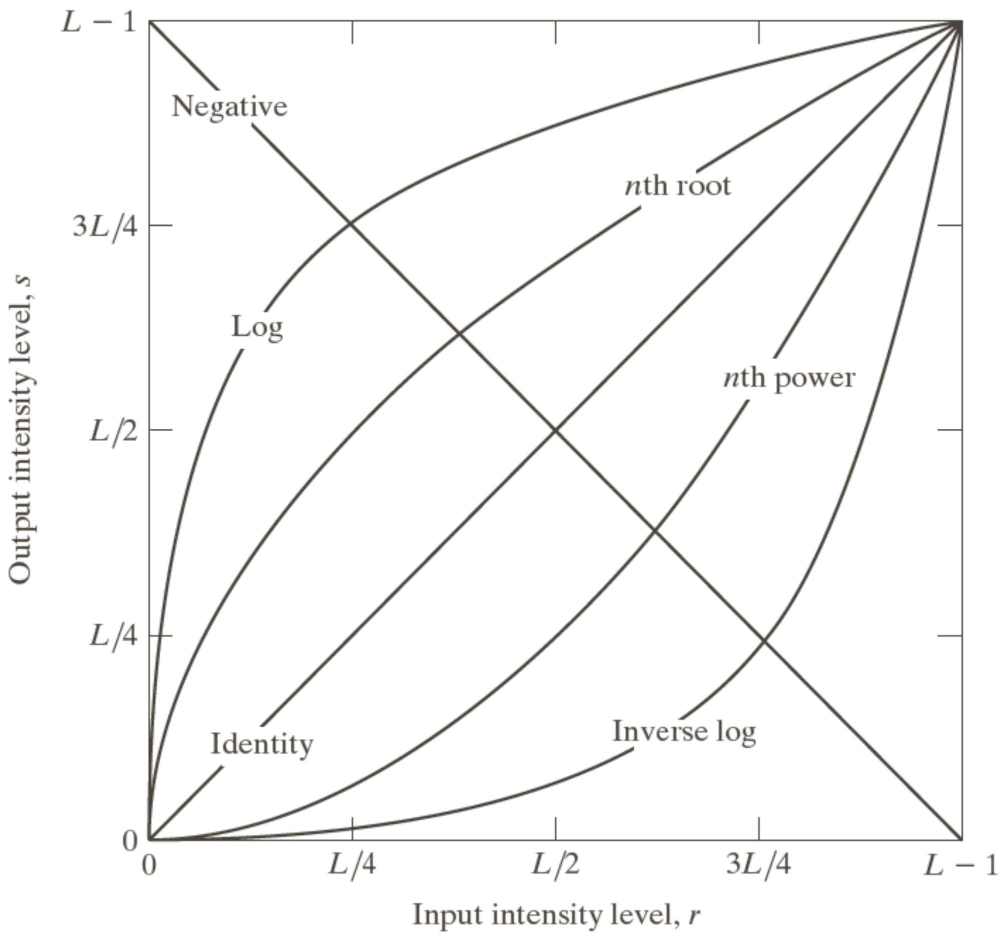


Image negatives

$s = L - 1 - r$

(c) Transfer Function of Butterworth High Pass filter

$$|H(u,v)|^2 = \frac{1}{1+\left[\dfrac{D_0}{D(u,v)}\right]^{2n}}$$

(d) Smoothing Linear filters

The general implementation for filtering an $M \times N$ image with a weighted averaging filter of size $m \times n$ is given

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

where $m = 2a+1, \quad n = 2b+1.$

(e) Laplacian in frequency domain

$$\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] = \boxed{-(u^2+v^2)}\, F(u,v)$$

$$H_1(u,v) = -(u^2+v^2)$$

Frequency domain

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Spatial domain

Laplacian operator

Ques.(2): Draw the block diagram of homomorphic filtering approach for image enhancement and explain it.

Homomorphic Filtering

ln : $z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$

DFT : $Z(u,v) = F_i(u,v) + F_r(u,v)$

H(u,v) : $S(u,v) = H(u,v)Z(u,v)$

(DFT)$^{-1}$: $s(x,y) = i'(x,y) + r'(x,y)$

exp : $g(x,y) = e^{s(x,y)} = i_0(x,y)r_0(x,y)$

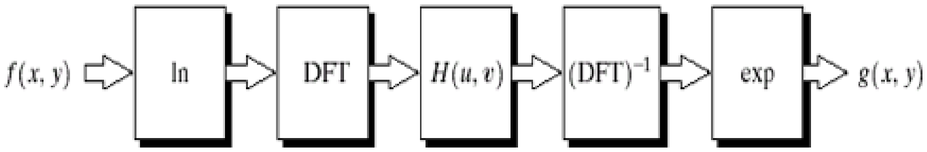
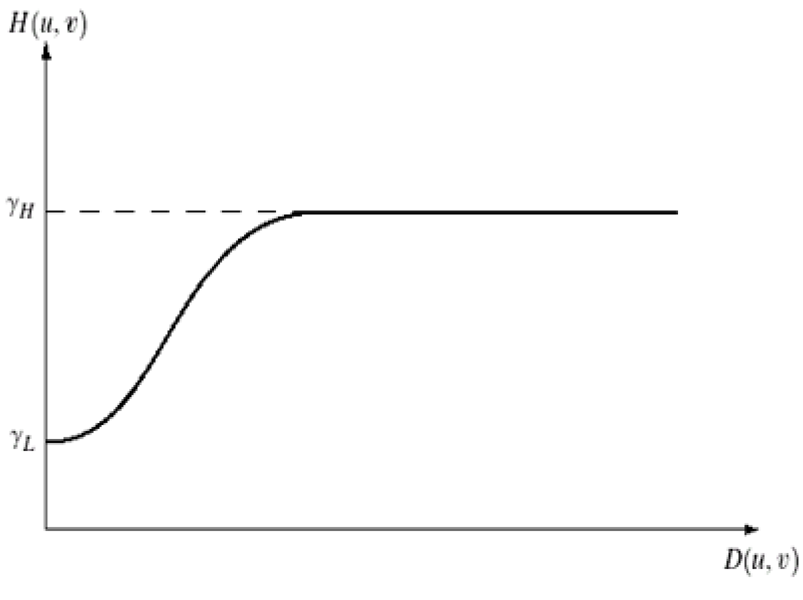


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

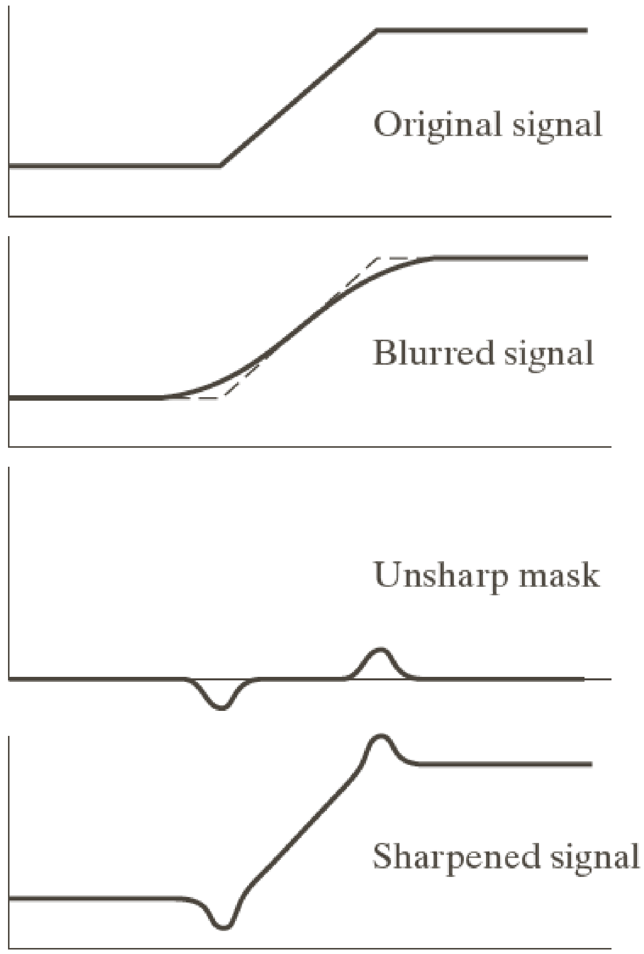


$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\left(\frac{D^2(u,v)}{D_0^2}\right)}\right] + \gamma_L$$

(OR)

Discuss high boost filtering for sharpening and Butterworth low pass filter for smoothing.

(5 marks)



Let $\bar{f}(x, y)$ denote the blurred image, unsharp masking is

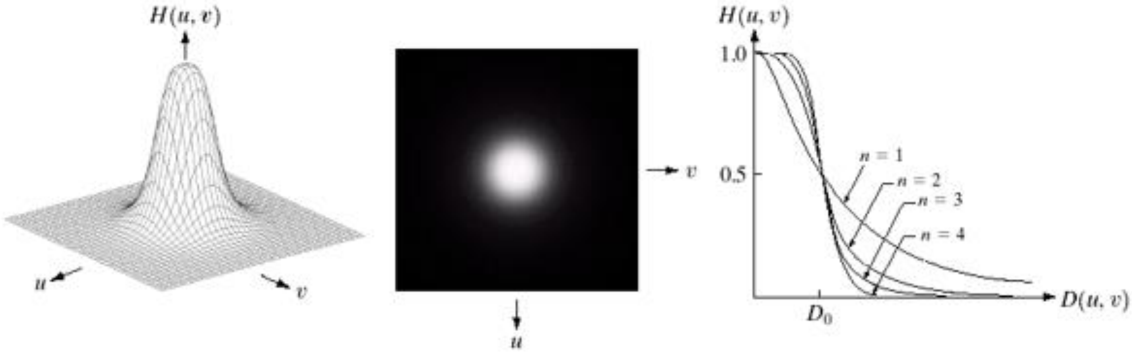
$$g_{mask}(x, y) = f(x, y) - \bar{f}(x, y)$$

Then add a weighted portion of the mask back to the original

$$g(x, y) = f(x, y) + k * g_{mask}(x, y) \qquad k \geq 0$$

when $k > 1$, the process is referred to as highboost filtering.

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$



Ques.(3): Explain Histogram Equalization **(5 marks)**

Histogram Equalization

The intensity levels in an image may be viewed as random variables in the interval [0, L-1].

Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s.

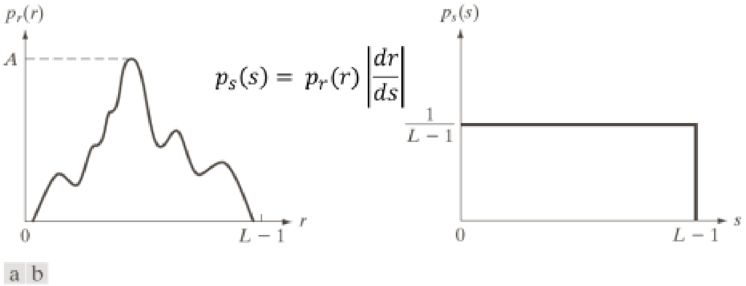


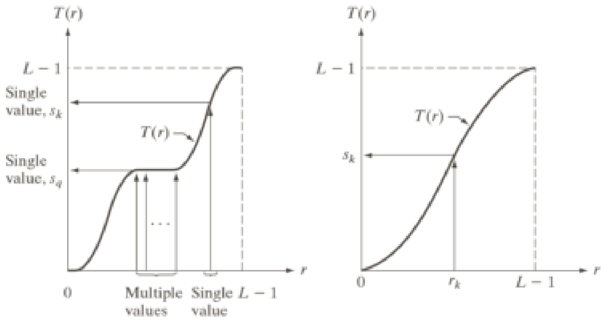
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization

$$s = T(r) \qquad 0 \le r \le L-1$$

a. T(r) is a strictly monotonically increasing function in the interval $0 \le r \le L\text{-}1$;

b. $0 \le T(r) \le L\text{-}1$ for $0 \le r \le L\text{-}1$.



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)dw\right]$$

$$= (L-1)\,p_r(r)$$

$$p_s(s) = \frac{p_r(r)dr}{ds} = p_r(r) \Big/ \left(\frac{ds}{dr}\right) = p_r(r) \Big/ \left((L-1)p_r(r)\right) = \frac{1}{L-1}$$

Histogram Equalization

Continuous case:

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Discrete values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= (L-1)\sum_{j=0}^{k}\frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k=0,1,\dots, L-1$$

Ques.(4): Explain image sharpening using Laplace Operator in both spatial and frequency domain

(4 Marks)

Sharpening Spatial Filters: Laplace Operator

The second-order isotropic derivative operator is the Laplacian for a function (image) f(x,y)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Sharpening Spatial Filters: Laplace Operator

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1
0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

Sharpening Spatial Filters: Laplace Operator

Image sharpening in the way of using the Laplacian:

$$g(x,y) = f(x,y) + c\left[\nabla^2 f(x,y)\right]$$

where,

$f(x,y)$ is input image,

$g(x,y)$ is sharpenend images,

$c = -1$ if $\nabla^2 f(x,y)$ corresponding to Fig. 3.37(a) or (b)

and $c = 1$ if either of the other two filters is used.

Laplacian in Frequency Domain

$$\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] = \boxed{-(u^2+v^2)}\,F(u,v)$$

$$H_1(u,v) = -(u^2+v^2)$$

Frequency domain

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \xrightarrow{\text{Spatial domain}} \text{Laplacian operator}$$

Laplacian operator

(OR)

Consider the image segment shown

(a) Let V = {0, 1} and compute the lengths of the shortest 4-, 8-, and m-path between p and q. If a particular path does not exist between these two points, explain why.

(b) Repeat for V = {1, 2}.

3	1	2	1 **(q)**
2	2	0	2
1	2	1	1
1 **(p)**	0	1	2