Mid Sem Electromagnetic field theory solution

Question 1. **I –** (c), **II –** (d)**, III –** (c), **IV –** (a) & (d)

Question 2.

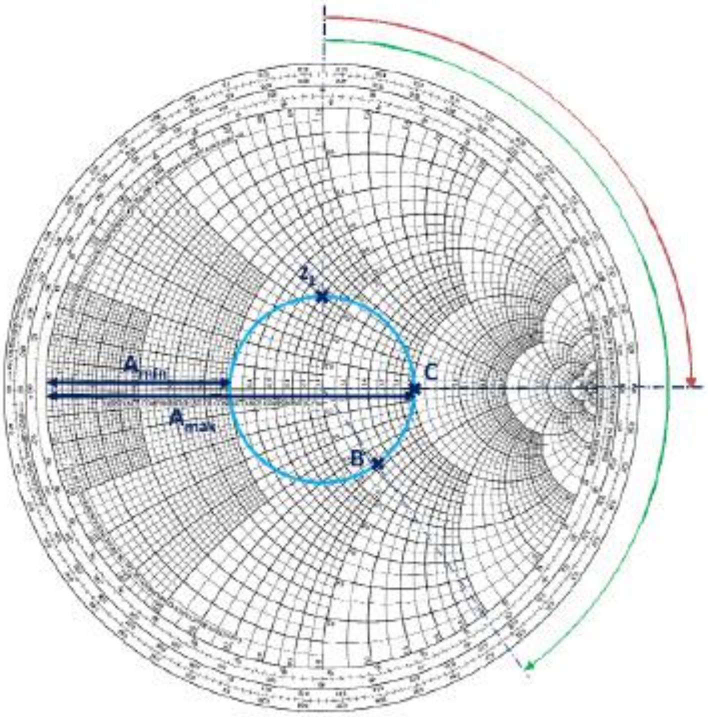
Solution-

$z_L = \frac{Z_L}{Z_o} = 0.8 + j0.6.$

a) $\Gamma = j0.333 = 0.333\angle 90°$, $S = \frac{A_{max}}{A_{min}} = 2$

b) Travel 0.2λ along green arrow to B: $z_{in} = 1.25 - j0.75 \Rightarrow Z_{in} = 62.5 - j37.5$ [Ω]

c) Travel along red arrow to Point C: $l = 0.25\lambda - 0.125\lambda = 0.125\lambda$; $z_{in} = 2 \Rightarrow Z_{in} = 100$ [Ω]



OR

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (1)$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (2)$$

$$\alpha = 0.04 \text{ dB/m} = \frac{0.04}{8.686} \text{ Np/m} = 0.00461 \text{ Np/m}$$

Multiplying (1) and (2),

$$Z_o(\alpha + j\beta) = R + j\omega L \longrightarrow 50(0.00461 + j2.5) = R + j\omega L$$

$$R = 50 \times 0.00461 = \underline{\underline{0.2305\ \Omega/\text{m}}}$$

$$L = \frac{50 \times 2.5}{2\pi \times 60 \times 10^6} = \underline{\underline{0.3316\ \mu\text{H/m}}}$$

Dividing (2) by (1),

$$\frac{\alpha + j\beta}{Z_o} = G + j\omega C$$

$$G = \frac{\alpha}{Z_o} = \frac{0.00461}{50} = \underline{\underline{92.2\ \ \mu\text{S/m}}}$$

$$C = \frac{\beta}{\omega Z_o} = \frac{2.5}{2\pi \times 60 \times 10^6 \times 50} = \underline{\underline{0.1326\ \ \text{nF/m}}}$$

Question 3.

Solution:

(a)
$$\rho_v = \nabla \bullet \boldsymbol{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 2(1+z^2) + 0 + 2x^2$$
$$= \underline{\underline{2(1+x^2+z^2)\ \ \text{nC/m}^3}}$$

(b)
$$\psi = \int_S D \bullet dS = \int_{y=0}^{3}\int_{x=0}^{2} 2x^2 z\,dx\,dy \bigg|_{z=1} = 2(1)\int_0^2 x^2 dx \int_0^3 dy$$
$$= 2\frac{x^3}{3}\bigg|_0^2 (3) = \underline{\underline{16\ \text{nC}}}$$

OR

(a)

$$\boldsymbol{J} = \nabla \times \boldsymbol{H} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi)\boldsymbol{a}_z = \frac{1}{\rho}\frac{\partial}{\partial \rho}(10^3 \rho^3)\boldsymbol{a}_z$$
$$= \underline{\underline{3\rho \times 10^3 \boldsymbol{a}_z\ \ \text{A/m}^2}}$$

(b)

Method 1:

$$I = \int_S \boldsymbol{J} \bullet d\boldsymbol{S} = \iint 3\rho\ \rho\, d\phi\, d\rho\, 10^3 = 3 \times 10^3 \int_0^2 \rho^2 d\rho \int_0^{2\pi} d\phi$$

$$= 3\times10^{3}\,(2\pi)\frac{\rho^{3}}{3}\Bigg|_{2}^{2} = 16\pi\times10^{3}\,A = \underline{\underline{50.265\text{ kA}}}$$

Method 2:

$$I = \oint_L \boldsymbol{H}\bullet d\boldsymbol{l} = 10^{3}\int_0^{2\pi}\rho^{2}\ \rho\, d\phi = 10^{3}(8)(2\pi) = \underline{\underline{50.265\text{ kA}}}$$

Question 4.

Solution: a)

$$\boldsymbol{B}_{2n} = \boldsymbol{B}_{1n} = 1.8\boldsymbol{a}_z$$

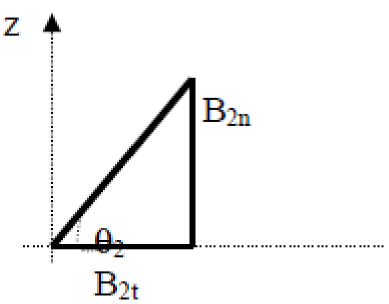
$$\boldsymbol{H}_{2t} = \boldsymbol{H}_{1t} \quad\longrightarrow\quad \frac{\boldsymbol{B}_{2t}}{\mu_2} = \frac{\boldsymbol{B}_{1t}}{\mu_1}$$

$$\boldsymbol{B}_{2t} = \frac{\mu_2}{\mu_1}\boldsymbol{B}_{1t} = \frac{4\mu_o}{2.5\mu_o}(6\boldsymbol{a}_x - 4.2\boldsymbol{a}_y) = 9.6\boldsymbol{a}_x - 6.72\boldsymbol{a}_y$$

$$\boldsymbol{B}_2 = \boldsymbol{B}_{2n} + \boldsymbol{B}_{2t} = 9.6\boldsymbol{a}_x - 6.72\boldsymbol{a}_y + 1.8\boldsymbol{a}_z \ \ \text{mWb/m}^2$$

$$\boldsymbol{H}_2 = \frac{\boldsymbol{B}_2}{\mu_2} = \frac{10^{-3}(9.6,-6.72,1.8)}{4\times4\pi\times10^{-7}}$$

$$= 1{,}909.86\boldsymbol{a}_x - 1{,}336.9\boldsymbol{a}_y + 358.1\boldsymbol{a}_z \ \ \text{A/m}$$



$$\tan\theta_2 = \frac{B_{2n}}{B_{2t}} = \frac{1.8}{\sqrt{9.6^2 + 6.72^2}} = 0.1536$$

$$\theta_2 = \underline{\underline{8.73^{o}}}$$

b)

$$J_d = \frac{\partial D}{dt} \longrightarrow D = \int J_d \, dt$$

$$D = \frac{-60\times10^{-3}}{109}\cos(10^9 t - \beta z)a_x = \underline{\underline{-60\times10^{-12}\cos(10^9 t - \beta z)a_x \text{ C/m}^2}}$$

$$\nabla\times E = \mu\frac{\partial H}{\partial t} \longrightarrow \nabla\times\frac{D}{\varepsilon} = -\mu\frac{\partial H}{\partial t}$$

$$\nabla\times\frac{D}{\varepsilon} = \frac{1}{\varepsilon}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_x & 0 & 0 \end{vmatrix} = \frac{1}{\varepsilon}(-60)(-1)\times10^{-12}\sin(10^9 t - \beta z)a_x$$

$$= \frac{60\beta}{\varepsilon}\times10^{-12}\sin(10^9 t - \beta z)a_y$$

$$H = -\frac{1}{\mu}\int\nabla\times\frac{D}{\varepsilon}dt = -\frac{1}{\mu}(-1)\frac{60\beta}{\varepsilon}\times\frac{10^{-12}}{10^9}\cos(10^9 t - \beta z)a_y$$

$$= \underline{\frac{60\beta}{\mu\varepsilon}\times10^{-21}\cos(10^9 t - \beta z)a_y \text{ A/m}}$$

OR

a)

Let x > 0 be region 1 and x < 0 be region 2.

$$D_{1n} = 50a_x, \quad D_{1t} = 80a_y - 30a_z$$

$$D_{2n} = D_{1n} = 50a_x$$

$$E_{2t} = E_{1t} \longrightarrow \frac{D_{2t}}{\varepsilon_2} = \frac{D_{1t}}{\varepsilon_1}$$

$$D_{2t} = \frac{\varepsilon_2}{\varepsilon_1}D_{1t} = \frac{7.6}{2.1}(80a_y - 30a_z) = 289.5a_y - 108.6a_z$$

$$D_2 = D_{2t} + D_{2n} = \underline{50a_x + 289.5a_y - 108.6a_z \text{ nC/m}^2}$$

b) Given, $E = 50\sin(10^8 t - 2z)$

i) From the above given expression of electric field, the direction of wave propagation is in +ve z-direction.

ii) $\omega = 10^8 \, rad/\sec$, β=2 rad/m

$$\omega / \beta = c / \sqrt{\varepsilon_r \mu_r} = \frac{3X10^8}{\sqrt{\varepsilon_r}}$$

$$\Rightarrow \sqrt{\varepsilon_r} = \frac{3\,X\,10^8\,X\,2}{10^8} = 6 \qquad \therefore \varepsilon_r = 36$$

$$f = \omega / 2\pi = \frac{10^8}{2\pi} = 15.92 \text{ MHz}$$

$$\lambda = \frac{2\pi}{\beta} = \pi = 3.14\ m$$

Question 5.

Solution:

$$\eta_1 = \eta_o, \qquad \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \eta_o / 2$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o / 2 - \eta_o}{3\eta_o / 2} = \underline{\underline{-1/3}}, \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{\eta_o}{3\eta_o / 2} = \underline{\underline{2/3}}$$

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1/3}{1 - 1/3} = \underline{\underline{2}}$$

$$\text{(b)} \quad E_{or} = \Gamma E_{oi} = -\frac{1}{3} \times (30) = -10$$

$$\boldsymbol{E}_r = -10 \cos(\omega t + z) \boldsymbol{a}_x \quad \text{V/m}$$

$$\text{Let} \quad \boldsymbol{H}_r = H_{or} \cos(\omega t + z) \boldsymbol{a}_H$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow -\boldsymbol{a}_x \times \boldsymbol{a}_H = -\boldsymbol{a}_z \longrightarrow \boldsymbol{a}_H = \boldsymbol{a}_y$$

$$\boldsymbol{H}_r = \frac{10}{120\pi} \cos(\omega t + z) \boldsymbol{a}_y = \underline{\underline{26.53 \cos(\omega t + z) \boldsymbol{a}_y}} \quad \text{mA/m}$$