Solution

4th - Semester

Numerical method and Computational tech.

Sol^n :- 1.

Let $f(x) = x^3 - x - 1$

and therefore $f(1) = 1^3 - 1 - 1 = -1 < 0$

$f(2) = 2^3 - 2 - 1 = 5 > 0$

$\therefore$ $[1, 2]$ contains a root of eq^n $f(x) = 0$

Now by Bisection method the first approximate root $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$

$$f(x_1) = (1.5)^3 - (1.5) - 1$$
$$= 0.875 > 0$$

Therefore, roots lies between 1 & 1.5.

$\therefore$ We take 2nd approximate value

$$x_2 = \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$$

$$f(x_2) = (1.25)^3 - (1.25) - 1$$
$$= -0.296875 < 0$$

therefore roots lies between x_2 & x_1

then 3rd approximate value

$$x_3 = \frac{1.25 + 1.5}{2} = 1.375$$

Therefore $f(x_3) = (1.375)^3 - 1.375 - 1$

$$= 0.2246 > 0$$

$\therefore$ roots lies between x_2 & x_3

And we take

$$x_4 = \frac{x_2 + x_3}{2} = \frac{1.25 + 1.375}{2}$$

$$= 1.3125$$

$$f(x_4) = (1.3125)^3 - (1.3125) - 1$$
$$= -.051520$$

$\therefore$ Roots lies between x_4 & x_3

we take

$$x_5 = \frac{1.3125 + 1.375}{2} = \frac{2.6875}{2}$$

$$= 1.34375$$

$$f(x_5) = .0826$$

$\therefore$ Roots lies between x_4 & x_5

$$x_6 = \frac{1.34375 + 1.3125}{2}$$

$$= \cancel{1.328125} \; 1.328125$$

$$f(x_6) = (1.328125)^3 - (1.328125) - 1$$
$$= 0.763916$$

$\therefore$ Roots lies between x_4 and x_6

$$x_7 = \frac{1.3125 + 1.328125}{2}$$

$$= 1.3203125$$

$$f(x_7) = -0.01871$$

$\because$ Roots lies between x_6 & x_7

$$x_8 = \frac{1.3203125 + 1.328125}{2}$$

$$= 1.324219$$

$$f(x_8) = -0.00212688$$

$\because$ Roots lies between x_6 & x_8

$$x_9 = \frac{1.328125 + 1.324219}{2}$$

$$= 1.326172$$

$$f(x_9) = 0.006209$$

Roots lies between x_9 & x_8

$$x_{10} = \frac{1.324219 + 1.326172}{2}$$

$$= 1.3251955$$

$$f(x_{10}) = 0.00203745$$

Roots lies between x_8 & x_{10}

$$x_{11} = \frac{1.324219 + 1.3251955}{2}$$

$$= 1.324707$$

$$f(x_{11}) = -0.000046728$$

Roots lies between x_{10} & x_{11}

$$x_{12} = \frac{1.3251955 + 1.324707}{2}$$

$$= 1.32495125$$

Solution :- 2(i).

Let $x = \sqrt{N}$

then $x^2 = N$

and $x^2 - N = 0$

therefore $f(x) = x^2 - N$

$f'(x) = 2x$

therefore by Newton Raphson's method if x_n denotes n^{th} itteration then

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^2 - N)}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

$$= \frac{x_n^2 + N}{2x_n}$$

$$= \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

where $n = 0, 1, 2 \ldots$

Sol^n :- 2(ii). Since $2 < \sqrt{8} < 3$ therefore we take $x_0 = 2.5$

Let $x = \sqrt{8}$

and we know that the itterative formula for a square root of 8. is

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

Here $N = 8$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{8}{x_n}\right]$$

where $n = 0, 1, 2 \ldots$

$x_1 = 2.85$

$x_2 = 2.8285$

$x_3 = 2.8284$

$x_4 = 2.8284$

3. Forward diff. Table

Sol^n :-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
2.0	9.0	1.06	0.13	$-.01$	.02
2.25	10.06	1.19	0.12	.01	
2.50	11.25	1.31	0.13		
2.75	12.56	1.44			
3.00	14.00				

Since $x_2 < x < x_3$, $u = \frac{2.35 - 2.25}{.25}$

$2.25 < 2.35 < 2.50$

$= \frac{.10}{.25} = .4$

$y(2.35) =$

$10.06 + (.4)(1.19) + \frac{(.4)(.4-1)(0.12)}{2!}$

$+ \frac{(.4)(.4-1)(0.01)(.4-2)}{3!}$

$= 10.06 + .476 + 0.0144 + .00064$

$= 10.53664 - 0.0144$

$= 10.52224$

Sol^n :- 4(i). Here $f(x) = \frac{1}{1+x^2}$

1. Divide the given interval into 6 equal sub intervals. of width $h = \frac{1-0}{6} = \frac{1}{6}$

$\therefore$ intervals are $(0, 1/6)$ $(1/6, 1/3)$ $(1/3, 1/2)$ $(1/2, 2/3)$ $(2/3, 5/6)$ $(5/6, 1)$

$x_0 = 0$ $\quad y_0 = 1$

$x_1 = 1/6$ $\quad y_1 = .97297$

$x_2 = 1/3$ $\quad y_2 = 0.9$

$x_3 = 1/2$ $\quad y_3 = 0.8$

$x_4 = 2/3$ $\quad y_4 = 0.6923$

$x_5 = 5/6$ $\quad y_5 = 0.5902$

$x_6 = 1$ $\quad y_6 = 0.5$

$$\int_0^1 \frac{1}{1+x^2}dx = \frac{1}{6\times 3}\left[(1+0.5)+4(0.9729+0.8+0.5902)\right.$$
$$\left.+2(0.9+0.6923)\right] = 0.7854$$

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Sol^n:- 4(ii) $f(x, y) = 3x^2 + 1$

Since $y(1) = 2$

$\therefore x_0 = 1, \ y_0 = 2$

$x_1 = x_0 + h = 1.5$

$x_2 = x_0 + 2h = 2$

Since we know that general recurrence relation of Euler's method

$$y_{i+1} = y_i + h f(x_i\ y_i) \qquad (1)$$

$$i = 1$$

$$y_2 = y_1 + h f(x_1 y_1)$$

$$y(x_2) = y(x_1) + h f(x_1, y_1) \qquad (2)$$

at $x = x_1$ putting $i = 0$

$$y_1 = y_0 + h f(x_0, y_0)$$

$$= 2 + 0.5 \times 4$$

$$= 2 + 2$$

$$f(x, y) = 3x^2 + 1$$

$$f(x_0 y_0) = 3 \times 1^2 + 1$$

$$= 4$$

$$y(1.5) = 4$$

∴ from relation (2)

$$y(x_2) = 4 + 0.5 \times (3 \times [(1.5)^2 + 1]$$

$$= 4 + 0.5 \times (3 \times 2.25 + 1)$$

$$= 7.875$$

Soln:- 5 (i).

The augemted matrix of the system is

$$M = \left[\begin{array}{ccc|c} 5 & 1 & 0 & 9 \\ -1 & 5 & -1 & 4 \\ 5 & -1 & 0 & -6 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_1$, $R_2 \rightarrow 5R_2$.

$$\sim \left[\begin{array}{ccc|c} 5 & 1 & 0 & 9 \\ -5 & 25 & -5 & 20 \\ 0 & -2 & 0 & -15 \end{array}\right]$$

$R_2 \rightarrow R_2 + R_1$

$$\sim \left[\begin{array}{ccc|c} 5 & 1 & 0 & 9 \\ 0 & 26 & -5 & 29 \\ 0 & -2 & 0 & -15 \end{array}\right]$$

$R_3 \rightarrow 13R_3$

$$\sim \left[\begin{array}{ccc|c} 5 & 1 & 0 & 9 \\ 0 & 26 & -5 & 29 \\ 0 & -26 & 0 & -195 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_2$

$$\left[\begin{array}{ccc|c} 5 & 1 & 0 & 9 \\ 0 & 26 & -5 & 29 \\ 0 & 0 & -5 & -166 \end{array}\right]$$

Reduced system is

$5x_1 + x_2 = 9$ —— (1)

$26x_2 - 5x_3 = 29$ —— (2)

$-5x_3 = -166$ —— (3)

From (3)

$$x_3 = \frac{166}{5} = 33.2$$

Now, from (2), we have

$$26x_2 = 29 + 5x_3 = 29 + 5 \times 166$$
$$= 859$$
$$x_2 = \frac{859}{26} = 33.0385$$

From (1),

$$5x_1 = 9 - x_2$$
$$= 9 - 33.0385$$
$$= -24.0385$$
$$x_1 = -4.8077$$

5(ii).

x_i	y_i	x_i^2	$x_i y_i$
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340

Let the straight line of best fit be

$$y = ax + b \quad \text{--- (1)}$$

The normal eq^n are

$$\Sigma y_i = a\Sigma x + 5b \quad \text{--- (2)}$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x \quad \text{--- (3)}$$

The values of Σx, Σy, Σx^2 and Σxy are calculated as

$$\Sigma x = 15, \quad \Sigma y = 204, \quad \Sigma x_i^2 = 55, \quad \Sigma xy = 748$$

$$204 = 15a + 5b \Rightarrow a = \frac{1}{15}(204 - 5b)$$

$$748 = 55a + 15b$$

$$748 = \frac{\cancel{55}^{11}}{\cancel{15}_3}(204 - 5b) + 15b$$

$$2244 = 2244 - 55b + 45b$$

$$b = 0, \quad a = \frac{204}{15} = 13.6$$

So, the straight line is $y = 13.6x$

Sol^n:- 6.

Error in Trapezoidal Rule :- ①

Expanding $y = f(x)$ in the neighbourhood of $x = x_0$ by Taylor's series, we get

$$y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \cdots \quad \text{—— (1)}$$

$$\therefore \int_{x_0}^{x_1} y\,dx = \int_{x_0}^{x_0+h} \left[y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \cdots\right]dx$$

$$= hy_0 + \frac{h^2}{2!}y_0' + \frac{h^3}{3!}y_0'' + \cdots \quad \text{—— (2)}$$

Now, area of the first trapezium in the interval $[x_0, x_1]$ is $A_1 = \frac{h}{2}(y_0 + y_1)$ —— (3)

Putting $x = x_0 + h$, $y = y_1$ in (1), we get

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \cdots \quad \text{—— (4)}$$

From (3) and (4), we get

$$A_1 = \frac{h}{2}\left[y_0 + y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \cdots\right]$$

$$= hy_0 + \frac{h^2}{2!}y_0' + \frac{h^3}{2.2!}y_0'' + \cdots \quad \text{—— (5)}$$

Therefore, the error in $[x_0, x_1]$ is

$$\int_{x_0}^{x_1} y\,dx - A_1 = \left(\frac{1}{3!} - \frac{1}{2.2!}\right)h^3 y_0'' + \cdots$$

$$\approx -\frac{h^3}{12}y_0'' \quad \text{(Neglecting other terms)}$$

Similarly, the error in $[x_1, x_2]$ is $-\frac{h^3}{12}y_1''$ ---- in $[x_{n-1}, x_n]$ is $-\frac{h^3}{12}y_{n-1}''$

Hence the total error is

$$E = -\frac{h^3}{12}(y_0'' + y_1'' + y_2'' + \cdots + y_{n-1}'')$$

Let $y''(\xi)$, $a < \xi < b$ be the maximum of $|y_0''|, |y_1''|, \ldots, |y_{n-1}''|$,

then, we have $E < -\frac{nh^3}{12}y''(\xi) = -\frac{(b-a)}{12}h^2 y''(\xi)$ $\quad \because h = \frac{b-a}{n}$

Hence the error in the Trapezoidal rule is of order h^2. $\quad n = \frac{b-a}{h}$