(1) (i) $y[n] = \dfrac{x[n]}{x[n-1]}$

a)

$x[n] \longrightarrow \boxed{S} \longrightarrow y[n] \longrightarrow \boxed{\text{Shifting}} \longrightarrow y[n-N_0]$

$$= \frac{x[n-N_0]}{x[n-1-N_0]} \quad \text{— (I)}$$

$x[n] \longrightarrow \boxed{\text{Shifting}} \longrightarrow x[n-n_0] \longrightarrow \boxed{\text{Syst.}} \longrightarrow y[n-N_0]$

$$= \frac{x[n-N_0]}{x[n-N_0-1]} \quad \text{— (II)}$$

Since both the equations (I) & (II) are same hence system is Time invariant system.

ii) b) $y[n] = n \cdot |x[n]|$

$x[n] \longrightarrow \boxed{S} \xrightarrow{y[n]} \boxed{\text{Shifting}} \longrightarrow y[n-N_0]$

$$= (n-N_0) \cdot |x[n-N_0]| \quad \text{— (I)}$$

$x[n] \longrightarrow \boxed{\text{Shift}} \xrightarrow{x[N-N_0]} \boxed{S} \longrightarrow y[N-N_0]$

$$= n \cdot |x[n-N_0]| \quad \text{— (II)}$$

Since both equations are not same, hence system is Time variant system.

ii) a) $y(t) = \frac{d}{dt} x(t)$; $x(t) = e^{-t} \cdot u(t)$

↳ unstable system; because differentiation of $x(t) = e^{-t}u(t)$ will result in to an impulse signal.

b) $y(t) = \frac{d}{dt} x(t)$; $x(t) = \sin t^2$

$= \frac{d}{dt}(\sin t^2) =$

$= 2t \cos t^2 \rightarrow$ <u>unstable</u>.

1 (iii) $y[n] = 3\sin[1.3\pi n + 0.5\pi] + 5\sin[1.2\pi n]$

$\omega_1 = 1.3\pi$

$\frac{\omega_1}{2\pi} = \frac{1.3\pi}{2\pi} = \frac{13}{20}$

= Ratio of integers.

$\omega_2 = 1.2\pi$

$\frac{\omega_2}{2\pi} = \frac{1.2\pi}{2\pi} = \frac{12}{20} = \frac{3}{5}$

= Ratio of integers

$\therefore \frac{K}{n_1} = \frac{13}{20}$

$n_1 = \frac{20K}{13}$

$\frac{K}{n_2} = 3/5$

$n_2 = \frac{5K}{3}$.

$\therefore$ Periodicity of $y[n]$

$= \text{LCM of } \left(\frac{20K}{13}, \frac{5K}{3}\right)$, $K$ = integer.

$= 20K$.

Hence $N_0 = 20$ samples.

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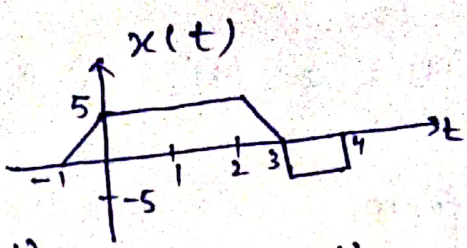
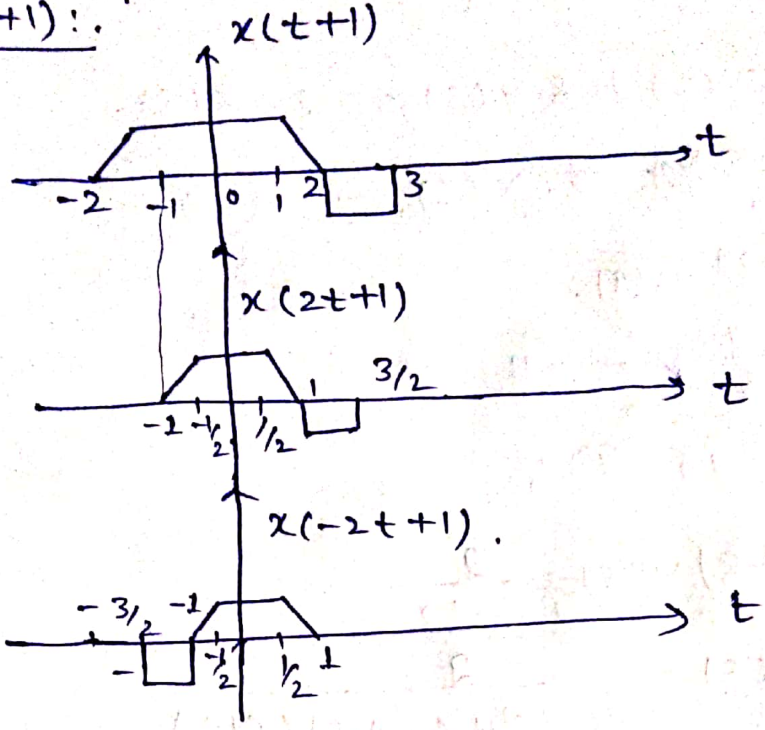
1 (iv)




Then $x(-2t+1)$ :




(V) let $x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 \, e^{-(t^2)/2} \, \delta(1-2t) \, dt$

$= \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} t^2 \cdot e^{-t^2/2} \cdot \delta\{-2(t-1/2)\} dt$

$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{2} \cdot (1/2)^2 \cdot e^{-\frac{(1/2)^2}{2}}$ . $\because$ (Impulse at $t = \frac{1}{2}$.)

$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot e^{-\frac{1}{8}}$

$= \frac{1}{8\sqrt{2\pi}\, e^{1/8}}$ Ans.

(2)(i) $x(t) = e^{-2t}u(t) \longleftrightarrow X(S)$.

a) $y_1(t) = X(2S)$.

$\because x(at) \xleftrightarrow{L.T} \frac{1}{|a|} X\left(\frac{S}{a}\right)$

or $|a| \cdot x(at) \longleftrightarrow X\left(\frac{S}{a}\right)$

or $X(S/a) \xleftrightarrow{I.L.T} |a| \cdot x(at)$, so put $a = 1/2$.

$\therefore X(2S) \longleftrightarrow \frac{1}{2} \cdot x(t/2)$. $\Rightarrow \boxed{y_1(t) = \frac{1}{2} \cdot e^{-t} u(t).}$

b) $y_2(t) = \frac{d}{ds} X(S)$

$\because -t \cdot x(t) \xleftrightarrow{L.T} \frac{d}{ds} X(S)$

$\therefore \frac{d}{ds} X(S) \longleftrightarrow -t \cdot x(t)$

$\therefore y_2(t) = -t \cdot e^{-2t} u(t)$

$\boxed{y_2(t) = -t e^{-2t} u(t).}$

c) $y_3(s) = S \cdot X(S)$

$\because \frac{d}{dt} x(t) \xleftrightarrow{L.T} S \cdot X(S)$

$\therefore y_3(t) = \frac{d}{dt} \cdot \{e^{-2t} u(t)\}$

$= -2e^{-2t} u(t) + e^{-2t} \cdot \delta(t)$

$\cancel{= (\delta(t) - 2u(t)) \cdot e^{-2t}}$. $[\because x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$

$\boxed{y_3(t) = \delta(t) - 2e^{-2t} u(t).}$

2. (ii) $F[z] = \dfrac{z}{(z+3)(z-4)}$, ROC: a) $3 < |z| < 4$

b) $|z| \cancel{<} 3$ & $|z| > 4$.

$$\frac{F[z]}{z} = \frac{1}{(z+3)(z-4)}$$

$$= \frac{A}{z+3} + \frac{B}{z-4}$$

~~As~~ From partial fraction;

$$A = -\frac{1}{7}, \quad B = \frac{+1}{7}$$

$$\frac{F(z)}{z} = \frac{-1/7}{z+3} + \frac{1/7}{z-4}$$

$$F(z) = \frac{-\frac{1}{7}z}{z+3} + \frac{\frac{1}{7}z}{z-4}$$

For ROC: a) $3 < |z| < 4$

$$f[n] = \frac{-1}{7} \cdot (-3)^n \cdot u[n] + \frac{1}{7} \cdot 4^n \cdot u[-n-1]$$

for ROC: b) $|z| > 3$ & $|z| > 4$.

~~f[n] =~~ ROC is Not common.

$\Rightarrow$ z-inverse transform doesn't exists.

2. (v). a) $y[n] = x[n] \cos \omega_0 n$

↳ Linear.

↳ static

↳ causal.

b) $y(t) = e^{x(t)} . \cos(x(t))$

↳ Non-linear

↳ static

↳ causal.

iii) $\dfrac{d^2y(t)}{dt^2} + 6\dfrac{dy(t)}{dt} + 8y(t) = 2x(t).$ ; $x(t) = e^{-2t}u(t).$

Taking the laplace transform,

$$s^2Y(s) + 6sY(s) + 8Y(s) = 2 \cdot X(s).$$

$$\Rightarrow \quad (s^2+6s+8)Y(s) = 2 \cdot X(s). \text{ —— (I)}$$

$$\because x(t) = e^{-2t}u(t),$$

$$\therefore X(s) = \frac{1}{s+2} \text{ —— (II)}$$

Putting $X(s)$ in (I),

$$(s^2+6s+8)Y(s) = \frac{2}{s+2}$$

$$\therefore Y(s) = \frac{2}{(s^2+6s+8)(s+2)}$$

$$= \frac{2}{(s+4)(s+2)(s+2)}$$

$$Y(s) = \frac{2}{(s+4)\cdot(s+2)^2}$$

$$\therefore Y(s) = \frac{A}{(s+4)} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)}$$

using partial fraction,

$$A = \frac{1}{2}, \quad B = 1, \quad C = -\frac{1}{2}$$

$$\therefore Y(s) = \frac{1/2}{s+4} + \frac{1}{(s+2)^2} \cancel{+} - \frac{1/2}{(s+2)}$$

$$y(t) = \frac{1}{2}e^{-4t}u(t) + \frac{1}{2}t \cdot e^{-2t}u(t) - \frac{1}{2}e^{-2t}u(t).$$

2(iv) $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$

$$= \left(\frac{1}{3}\right)^{-n} \cdot u[-n-1] + \left(\frac{1}{3}\right)^n \cdot u[n] - \left(\frac{1}{2}\right)^n u[n]$$

let $x_1[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$

$$\therefore X_1(z) = \sum_{-1}^{-\infty} \left(\frac{1}{3}\right)^{-n} \cdot z^{-n}$$

$$= \sum_{-1}^{-\infty} \left(\frac{z}{3}\right)^{-n}$$

$$= \frac{z}{3} + \left(\frac{z}{3}\right)^2 + (z/3)^3 + \cdots$$

$$= \frac{z/3}{1 - z/3}, \quad |z/3| < 1$$

$$= \frac{z}{3-z}, \quad |z| < 3.$$

let $x_2[n] = \left(\frac{1}{3}\right)^n u[n]$.

$\therefore X_2(z) = \frac{z}{z - 1/3}, \quad |z| > 1/3$ (By property)

$\because \left[ a^n u[n] \xleftrightarrow{Z.T} \frac{z}{z-a}, |z| > a \right]$

& let $x_3[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$.

$$\therefore X_3(z) = \frac{z}{z - 1/2}, \quad |z| > 1/2.$$

Since Z-transforms follows the linearity,

$\therefore X(z) = Z.T. \{ x_1[n] + x_2[n] + x_3[n] \}$

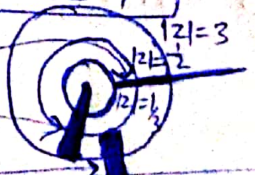
$$\boxed{X(z) = \frac{z}{3-z} + \frac{3z}{3z-1} - \frac{2z}{2z-1}},$$

& the common ROC will be,

For. $X_1(z)$, $|z| < 3$

$X_2(z)$, $|z| > 1/3$

$X_3(z)$, $|z| > 1/2$




$\therefore$ $\boxed{\text{common ROC } \frac{1}{2} < |z| < 3}$

3 For a system x(t), its Laplace transform is,

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

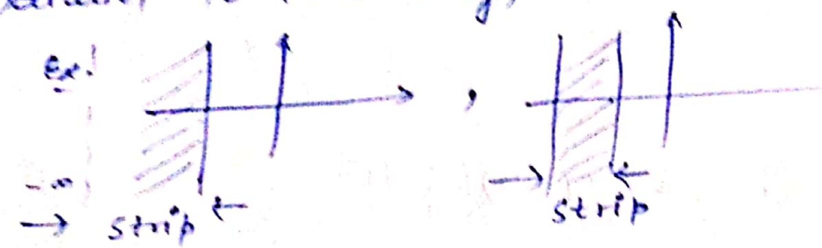
$$= \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt \quad (\text{as } s = \sigma + j\omega)$$

For this integral to be finite $\left| \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt \right| < \infty$, or $\left| \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t}\, dt \right| < \infty$, as the amplitude of complex exponential be always unity.

For a fixed x(t), the whole convergence criteria depends on $\sigma$ i.e. (ROC) value & doesn't depend on '$\omega$'. It simply means $\omega$ can be any finite value. i.e. $-\infty < \omega < \infty$.

So if we plot ROC for any '$\sigma$' it will be a strip parallel to vertical (y) axis.

Ex:




* For a sequence x[n] the z-transform is defined as.

$$X(z) = \sum_{-\infty}^{\infty} x[n]\, z^{-n}$$

$$= \sum_{-\infty}^{\infty} x[n] \cdot r^{-n} \cdot e^{-j\omega n} \quad \text{as } (z = re^{j\omega} = r\angle\omega,\ r \text{ is radius \& } \omega \text{ is angle.}$$

For this summation to be finite, $X(z) = \left| \sum_{-\infty}^{\infty} x[n] \cdot r^{-n} \cdot e^{-j\omega n} \right| < \infty$

or $\left| \sum_{-\infty}^{\infty} x[n] \cdot r^{-n} \right| < \infty$, as the amplitude of complex exponential be always unity.

So for a fixed x[n], the whole criteria depends on value of r which is radius & doesn't depend on '$\omega$' so it can be concluded that '$\omega$' can be any angle from 0 to $2\pi$.

Hence if we plot ROC for z-transform it will be a circle of radius 'R'.

Ex.