Q.1. Solution :-

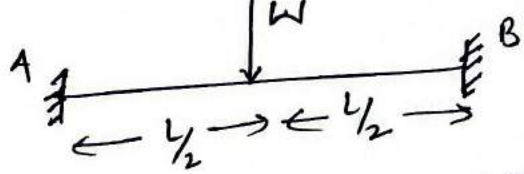
(a) ———— (iv) Flexibility matrix method

(b) ———— (iv) None of these

(c) ———— (iii) 40 kN-m

Explakation

Let, W kN Load at mid span



Now, Fixed end moment

$$\frac{WL}{8} = 60\,kN\text{-}m$$

$$\therefore W = \frac{60 \times 8}{L}\,kN$$

when we distributed it over whole span as udl

then $w = \frac{W}{L} = \frac{60 \times 8}{L^2}\,kN/m$

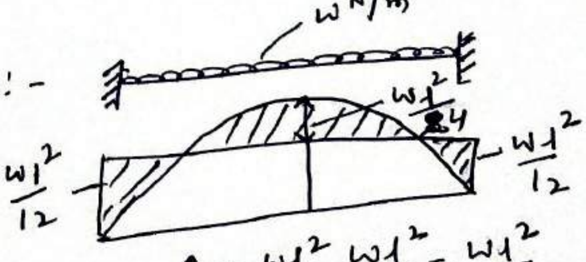
Now For that Fixed end moment

$$= \frac{wL^2}{12} = \frac{\cancel{60}^{20} \times \cancel{8}^{2} \times \cancel{L^2}}{\cancel{12}_{\cancel{3}} \times \cancel{L^2}}$$

$$= 40\,kN\text{-}m.$$

(d) ———— (iv) $\frac{wl^2}{24}$

Explanation:-



$$\updownarrow = \frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24}$$

(e) ———— (iv) $\frac{8EI}{L}$

Q.2. solution :-

Calculation of fixed end moment :-

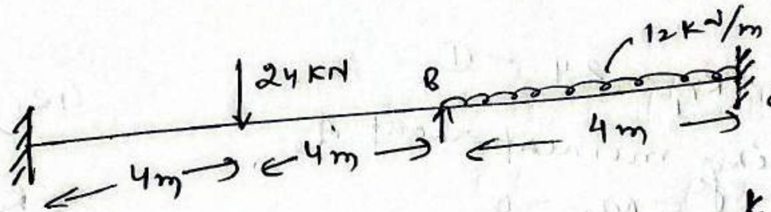
$$M_{FAB} = -\frac{WL}{8} = \frac{-24 \times \cancel{8}}{\cancel{8}} = -24\ KNm$$

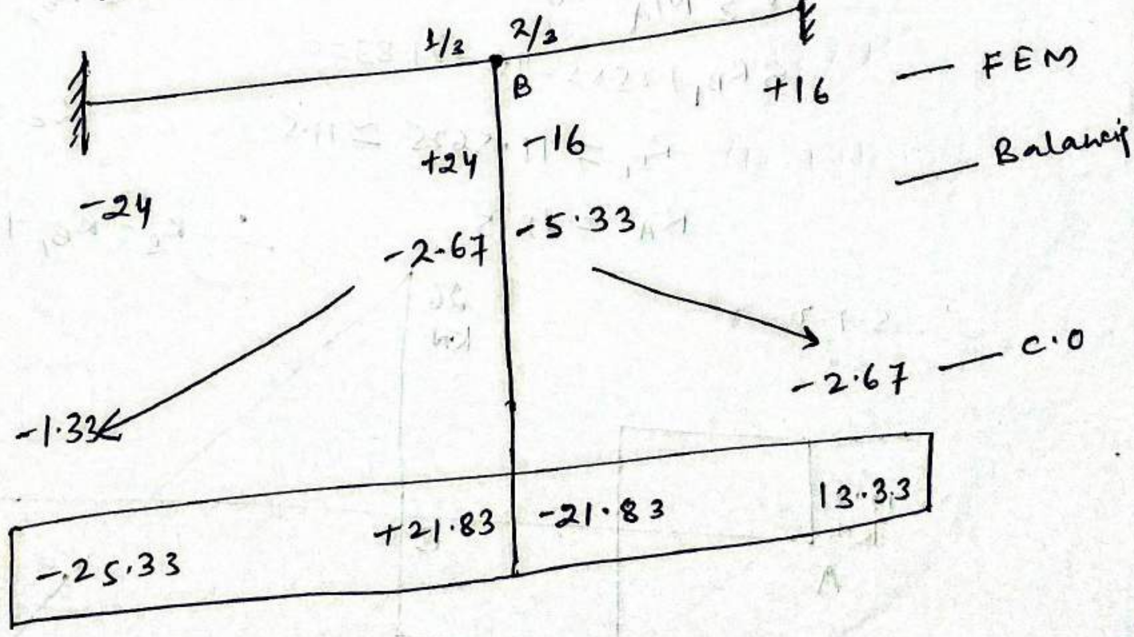
$$M_{FBA} = +\frac{WL}{8} = \frac{+24 \times \cancel{8}}{\cancel{8}} = 24\ KN\text{-}m$$

$$M_{FBC} = -\frac{WL^2}{12} = \frac{-\cancel{12} \times 4^2}{\cancel{12}} = -16\ KN\text{-}m$$

$$M_{FCB} = +\frac{WL^2}{12} = \frac{+\cancel{12} \times 4^2}{\cancel{12}} = +16\ KN\text{-}m$$

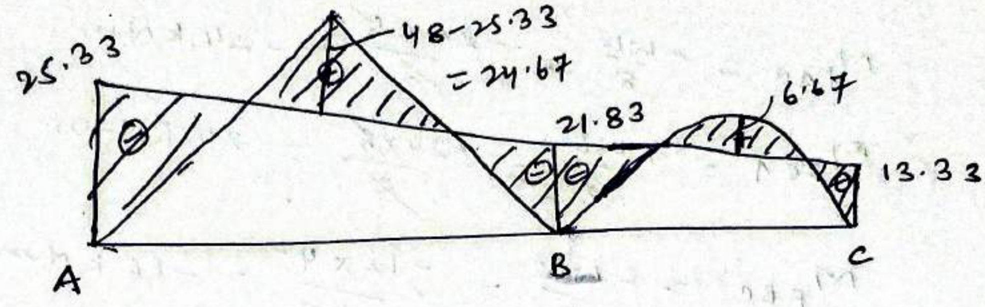
Joint	Member	Stiffness	Total Stiffness	Distribution factor
B	BA	$4EI/8$	$3EI/2$	$1/3$
	BC	$4EI/4$		$2/3$





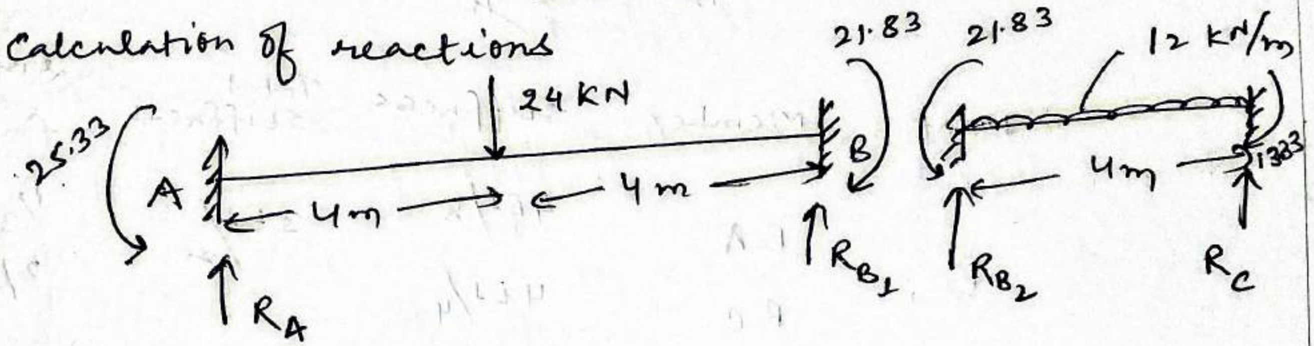
Final end moment

Hence superimposing free bending moment over fixed end moment net we get.



B.M.D

Calculation of reactions



$R_A + R_{B_1} = 24$ —(1)

Taking moment about A,

$\Sigma M_A = 0$

$8R_{B_1} + 25.55 - 96 - 21.83 = 0$

$R_{B_1} = 11.5625 \simeq 11.5$

$R_A = 12.5$

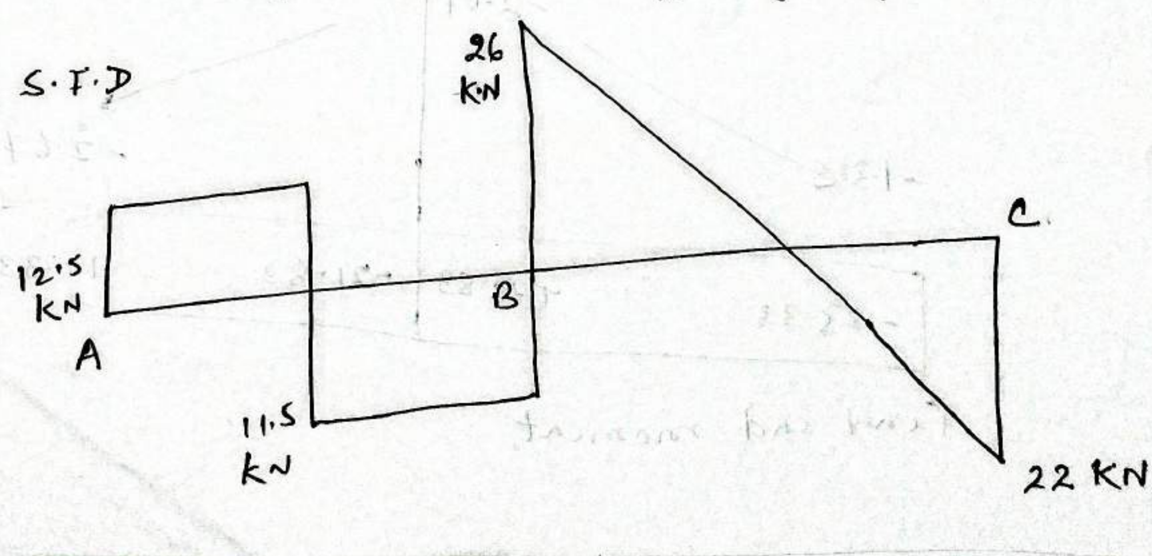
$4R_{B_2} + 13.33 - 21.83 - 96 = 0$

$\therefore R_{B_2} = 26.125 \simeq 26$

$R_C = 22\text{ kN}$.

$\therefore R_B = R_{B_1} + R_{B_2} = 37.5\text{ kN}$

$\therefore$ S.F.D



Q.3. Solution:-

No, horizontal sway because hinge at 'C'.

Calculation of fixed end moments

$$M_{FAB} = \frac{-WL}{8} = -\frac{\overset{15}{\cancel{30}} \times \cancel{4}}{\cancel{8}_2} = -15 \text{ kNm}$$

$$M_{FBA} = +\frac{WL}{8} = \frac{+30 \times 4}{8} = +15 \text{ kNm}$$

$$M_{FBC} = -120 \times \frac{4}{8} = -60 \text{ kNm}$$

$$M_{FCB} = +120 \times \frac{4}{8} = +60 \text{ kNm}$$

Now slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(\cancel{2\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$\because \theta_A = 0$

$\Delta = 0$

$$M_{AB} = -15 + \frac{2EI}{4}\theta_B$$

$$M_{BA} = +15 + \frac{2EI}{4}(2\theta_B)$$

$$M_{BC} = -60 + \frac{2 \times 2EI}{4}(2\theta_B + \theta_C)$$

$$M_{CB} = +60 + \frac{2 \times 2EI}{4}(2\theta_C + \theta_B)$$

Using equilibrium equation

due to hinge at C, $M_{CB} = 0$

or $\theta_B + 2\theta_C = \frac{-60}{EI}$ —(1)

At joint B,

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow \quad EI\theta_B + 15 + 2EI\theta_B + EI\theta_c - 60 = 0$$

$$\Rightarrow \quad 3\theta_B + \theta_c = \frac{45}{EI} \quad \text{—— (ii)}$$

Solving equation (i) and (ii) we get

$$\theta_B = \frac{30}{EI} \quad \text{(clockwise)}$$

$$\theta_C = \frac{-45}{EI} \quad \text{(Anticlockwise)}$$

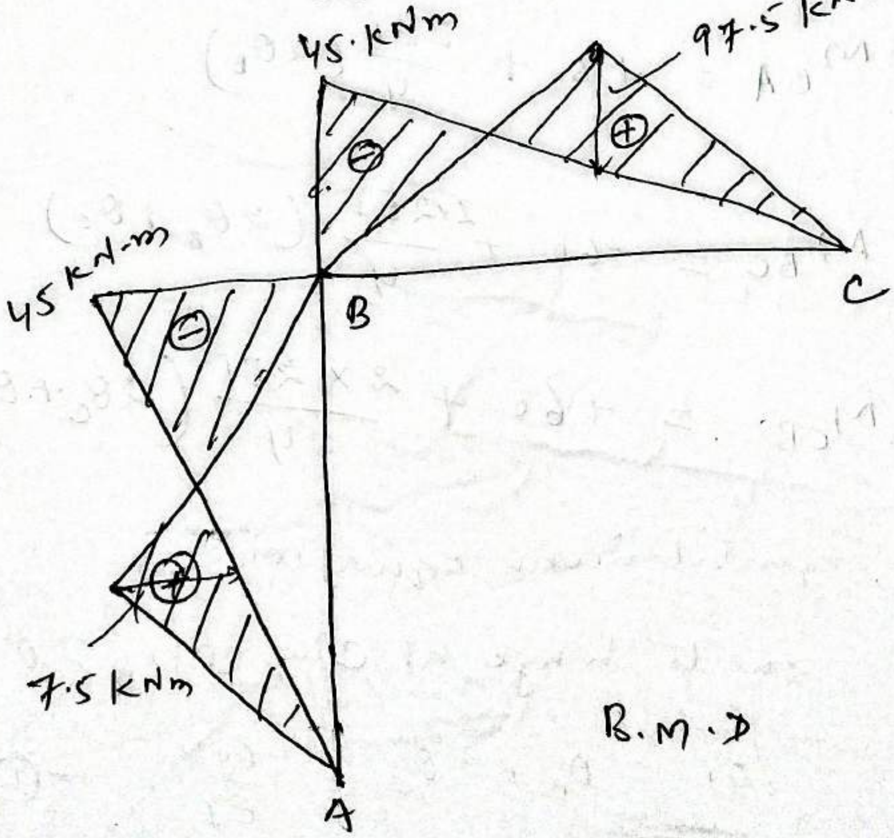
Now using the value of θ_B and θ_c we have

$$M_{AB} = 0$$

$$M_{BA} = 45\,kNm$$

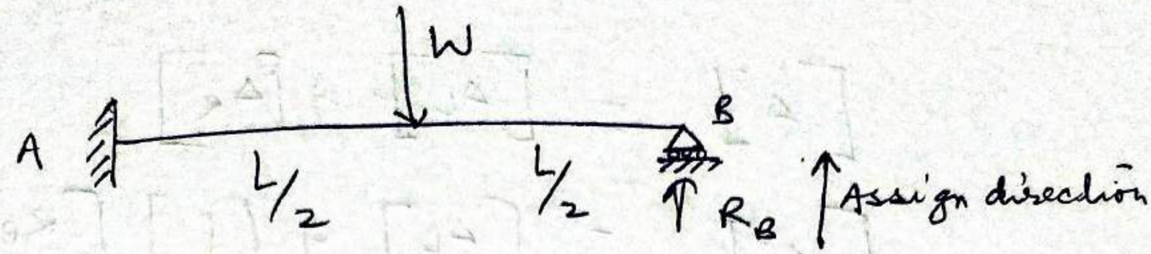
$$M_{BC} = -45\,kNm$$

$$M_{CB} = 0$$



B.M.D

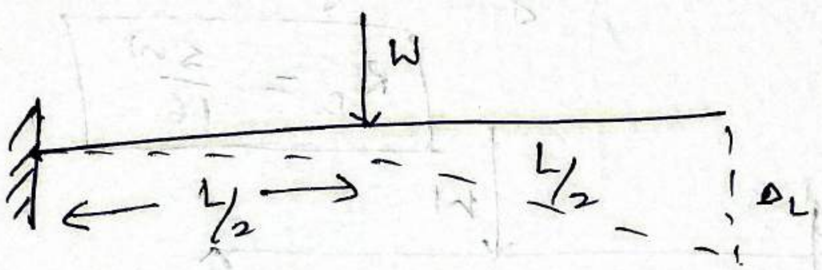
Q.4. Solution:-



Here degree of static indeterminacy = 1

Consider it R_B.

Now released structure



$$\Delta_L = -\left\{\frac{W\left(\frac{L}{2}\right)^3}{3EI} + \frac{W\left(\frac{L}{2}\right)^2}{2EI}\times\frac{L}{2}\right\}$$

Δ_L → displacement due to external load at the assign Co-ordinate

$$= -\left\{\frac{WL^3}{24EI} + \frac{WL^3}{16EI}\right\}$$

$$= -\frac{5WL^3}{48EI}$$

Now due to redundent 'R_B' displacement in assign direction/Co-ordinate

$$[\Delta_R] = \frac{R_B L^3}{3EI} = [f]\,[R_B]$$

$$= \left[\frac{L^3}{3EI}\right][R_B]$$

Now, using Compatibility equation

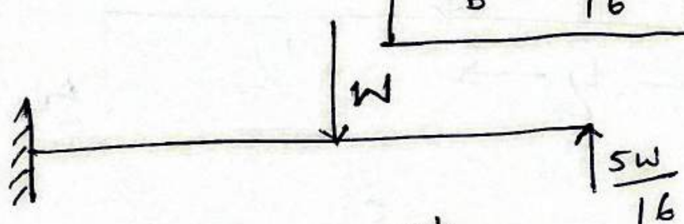
$$[\Delta] = [\Delta_L] + [\Delta_R]$$

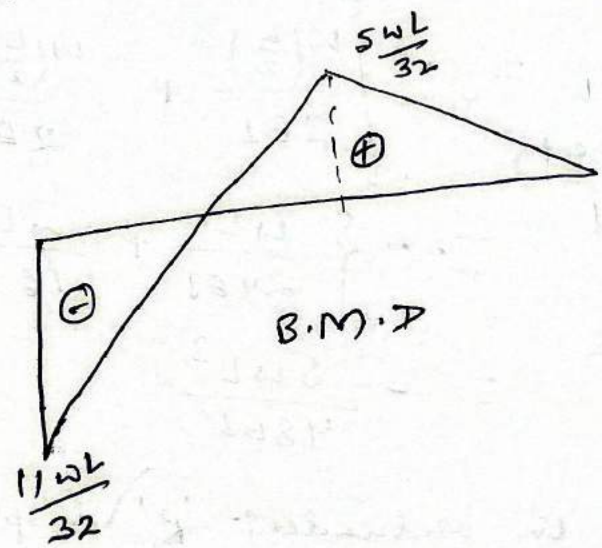
$$[\Delta] = [\Delta_L] + [f][R_B]$$

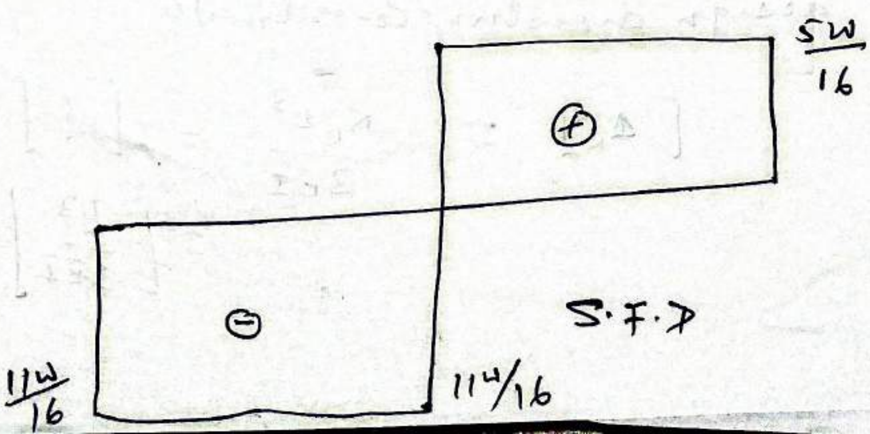
$$\Rightarrow \quad 0 = -\frac{5WL^3}{48EI} + \left[\frac{L^3}{3EI}\right][R_B]$$

Solving we get

$$\boxed{R_B = \frac{5W}{16}}$$

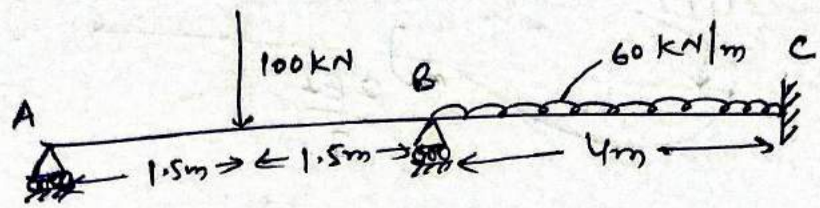






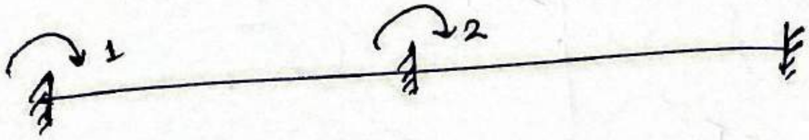
Q.5. solution

Given,

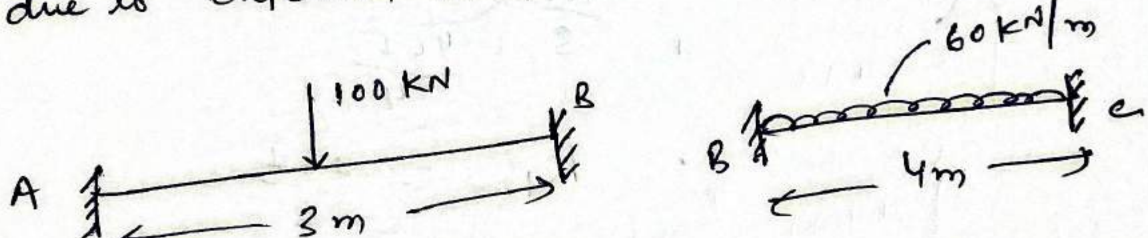


D.O.F = 2 θ_A and θ_B

Assign Co-ordinate as.



Now forces [P'] in the locked structure due to external load



$$P_1' = M_{FAB} = \frac{-WL}{8} = \frac{-100\times 3}{8} = -37.5 \text{ kNm}$$

$$P_2' = +M_{FBA} - M_{FBC} = \frac{100\times 3}{8} - \frac{60\times 4^2}{12}$$

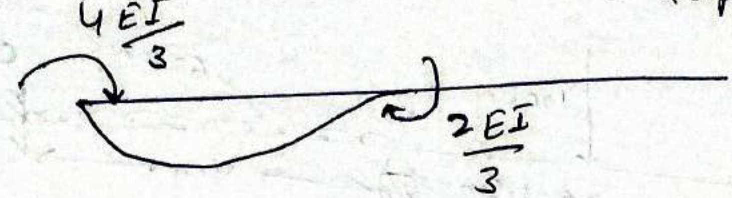
$$= 37.5 - 80$$

$$= -42.5 \text{ kNm}$$

$$[P'] = \begin{bmatrix} -37.5 \\ -42.5 \end{bmatrix}$$

Now development of stiffness matrix.

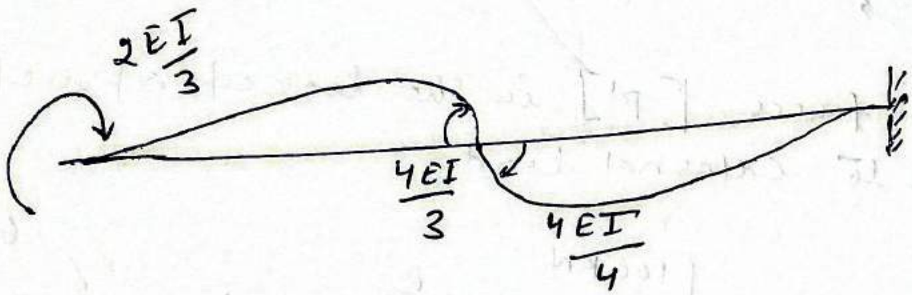
If $\Delta_1 = 1$ and $\Delta_2 = 0$ {i.e provide unit displacement in ordinate 1, keep $\Delta_2 = 0$}



$$\therefore \; K_{11} = 4EI/3$$

$$K_{21} = \frac{2EI}{3}$$

Now, $\Delta_2 = 1$ and $\Delta_1 = 0$



$$\therefore \; K_{12} = \frac{2EI}{3}$$

$$K_{22} = \frac{7EI}{3} \quad \left\{\because \frac{4EI}{3} + EI\right\} = 7EI/3$$

$$\therefore \; [K] = \begin{bmatrix} 4EI/3 & 2EI/3 \\ \frac{2EI}{3} & 7EI/3 \end{bmatrix}$$

Stiffness Matrix

Now using Condition of equilibrium

External forces at the Co-ordinate

= Force required to locked the Structure

+ Force Corresponding to unlocked structure

$$\Rightarrow \quad [P] = [P'] + [K][\Delta]$$

$$\Rightarrow \quad 0 = \begin{bmatrix} -37.5 \\ -42.5 \end{bmatrix} + \begin{bmatrix} \frac{4EI}{3} & 2EI/3 \\ \frac{2EI}{3} & \frac{7EI}{3} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

or

$$4EI\,\Delta_1 + 2EI\,\Delta_2 = 112.5 \quad \text{—} (i)$$

$$2EI\,\Delta_1 + 7EI\,\Delta_2 = 127.5 \quad \text{—} (ii)$$

Solving eq^n (i) and (ii) we get

$$\Delta_1 = 22.1875/EI$$

$$\Delta_2 = 11.875/EI$$

Now using equations

$$M_{AB} = M_{FAB} + \frac{2EI}{3}(2\theta_A + \theta_B)$$

Here $\theta_A = \Delta_1$

$\theta_B = \Delta_2$

Putting value of Δ_1 and Δ_2 we get

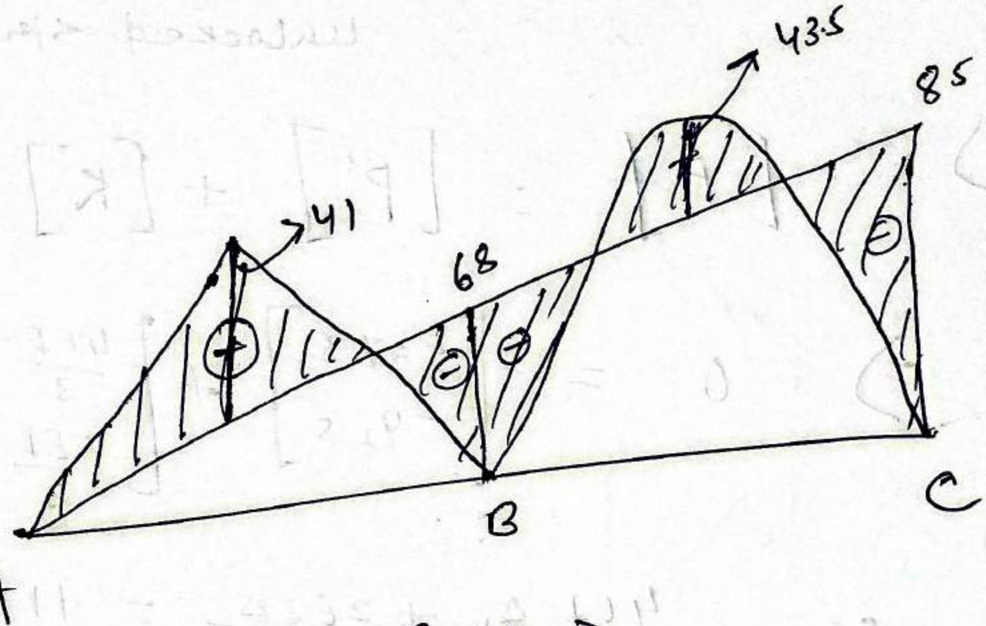
$M_{AB} = 0$

$M_{BA} = 68$ kN m

$M_{BC} = -68$ kN m

$M_{CB} = 85$ kNm

Now,



B.M.D